1. [**4**] Mr. Schwartz has 96 pringles and 120 pieces of candy. What is the largest number of students for which both pringles and candy can be split equally among them?

Proposed by Lewis Lau.

Answer: $\boxed{24}$

Solution: We want to find $\gcd(96, 120)$. $96 = 2^5 \cdot 3$ and $120 = 2^3 \cdot 3 \cdot 5$, so $\gcd(96, 120) = 2^3 \cdot 3 = 24$.

2. [**4**] It takes Gloria the Snail 40 hours to crawl around a rectangular basketball court and 46 hours to crawl around a rectangular tennis court, which has a perimeter 4 meters longer than the basketball court. If Gloria the Snail crawls at a constant speed, what is Gloria the Snail's speed in meters per hour?

Proposed by Reanna Jin.

Answer: $\boxed{\dfrac{2}{3}}$

Solution: It takes Gloria the Snail $46 - 40 = 6$ hours to crawl 4 meters, so her speed is $\frac{4}{6} = \frac{2}{3}$ meters per hour.

3. [**4**] Let $a \star b = \frac{a+b}{a}$. What is $7 \star (8 \star 7) - 8 \star (7 \star 8)$?

Proposed by Olivia Guo.

Answer: $\boxed{0}$

Solution: We first compute $a \star (b \star a)$:

$$a \star (b \star a) = \frac{a + \frac{b+a}{a}}{a} = \frac{ab + a + b}{ab}.$$

This is symmetric about a and b, so $a \star (b \star a) = b \star (a \star b)$, and $7 \star (8 \star 7) - 8 \star (7 \star 8) = 0$.

4. [**5**] Kite $ABCD$ is inscribed in a circle. If the area of the kite is 48 square units and BD is 6 units long, what is the area of the circle?

Proposed by Evan Zhang.

Answer: $\boxed{64\pi}$

Solution: Because $ABCD$ is a kite, $AC \perp BD$, so the area of $ABCD$ is $\frac{AC \cdot BD}{2} = 48$. Since $BD = 6$, $AC = 16$. Additionally, by symmetry, AC is a diameter of the circle, so the area of the circle is $\pi \cdot \left(\frac{16}{2}\right)^2 = 64\pi$.

5. **[5]** Valerie draws a right triangle with legs of length 1 and 8. Michelle draws a different right triangle with legs of integer length. To their surprise, the hypotenuses of both right triangles are the same length! What is the area of Michelle's right triangle?

Proposed by Jason Youm.

Answer: $\boxed{14}$

Solution: The hypotenuse of Valerie's triangle is $\sqrt{1^2 + 8^2} = \sqrt{65}$. We can find that $4^2 + 7^2 = 16 + 49 = 65$, so Michelle's triangle has legs of length 4 and 7. The area of Michelle's triangle is $\frac{1}{2} \cdot 4 \cdot 7 = 14$.

6. **[5]** If $1^3 + 2^3 + 3^3 + \cdots + n^3 = 2025$, what is n?

Proposed by Arjun Samavedam.

Answer: $\boxed{9}$

Solution: The formula for the sum of the first n cubes is $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$. We then have $\frac{n(n+1)}{2} = \sqrt{2025} = 45$, so $n(n+1) = 90$, giving $n = 9$.

7. **[6]** Olivia thinks that two plus two equals five. As in, she believes there are solutions to the following equation:

$$\begin{array}{r} \text{T W O} \\ + \quad \text{T W O} \\ \hline \text{F I V E} \end{array}$$

In Olivia's equation, each letter represents a distinct digit. What is the maximum possible value of $FIVE$?

Proposed by Ivy Guo.

Answer: $\boxed{1872}$

Solution: To maximize $FIVE$, let $T = 9$, so $F = 1$. Then, I is either 8 or 9, but since each letter represents a distinct digit, $I \neq 9$, so $I = 8$. Therefore, $W \leq 4$.

However, we also must have $W \neq 4$, because if $W = 4$, then V must be either 8 or 9, but both 8 and 9 have already been used. Therefore, the largest possible value of W is 3.

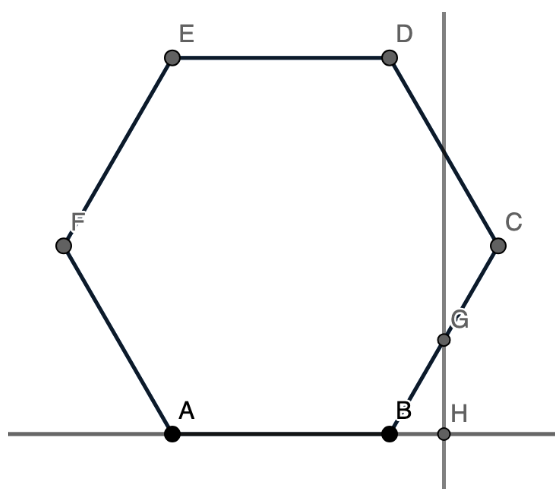
The largest digit left is 7. We can check that $O = 7$ does not work, because $937 + 937 = 1874$, and the digit 7 is used twice. $O = 6$ does work, and $936 + 936 = 1872$.

8. **[6]** Two ants start on the same vertex of a regular hexagon with side length 2 and begin running in opposite directions along the sides of the hexagon. If one ant runs 3 times as fast as the other, what is the distance from the point where they first meet to their starting location?

Proposed by Evan Zhang.

Answer: $\boxed{\sqrt{7}}$

Solution: The two ants will meet $\frac{1}{4}$ of the way around the hexagon. With a perimeter of $6 \cdot 2 = 12$, this is 3 units along the hexagon.



In this diagram, if the ants start at A, and the faster initially goes towards F and the slower one initially goes towards B, they will meet at G. $\triangle GBH$ is a $30-60-90$ triangle. As $BG = 1$, $BH = \frac{1}{2}$, and $GH = \frac{\sqrt{3}}{2}$. With the Pythagorean Theorem, $AG = \sqrt{\left(2 + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{7}$.

9. **[7]** What is the maximum number of intersection points between 3 ellipses and 3 lines?

Proposed by Evan Zhang.

Answer: $\boxed{33}$

Solution: To maximize the number of intersection points, no three curves (ellipses or lines) should intersect at one point. Any two ellipses can intersect with each other at at most 4 points, so there can be up to $3 \cdot 4 = 12$ ellipse-ellipse intersection points. Any two lines can intersect at at most 1 point, so there can be up to 3 line-line intersection points. A line and an ellipse can intersect at at most 2 points. There's $3 \cdot 3 = 9$ combinations of a line and an ellipse, for a total of 18 line-ellipse intersection points. The total number of intersection points is therefore $12 + 3 + 18 = 33$.

10. **[8]** If positive integers a, b, and c satisfy $\gcd(a, b) = 30$, $\gcd(b, c) = 18$, and $\gcd(c, a) = 24$, what is the minimum value of abc?

Proposed by Lewis Lau.

Answer: $\boxed{777600}$

Solution: The prime factorizations of the gcd's are $30 = 2^1 \cdot 3^1 \cdot 5^1$, $18 = 2^1 \cdot 3^2 \cdot 5^0$, and $24 = 2^3 \cdot 3^1 \cdot 5^0$. The exponents in the factorization of the gcd of two numbers are the smaller of the corresponding exponents in their prime factorizations. Looking at each exponent:

- 24 has the largest exponent for 2, being 3. As such, $2^3 = 8$ must divide both c and a. 2 and not 4 divides both 30 and 18, so 2 must divide b. The minimum exponent of 2 for a, b, and c are 3, 1, and 3, respectively.
- 18 has the largest exponent for 3, being 2. As such, $3^2 = 9$ must divide both b and c. 3 and not 9 divides both 30 and 24, so 3 must divide a. The minimum exponent of 3 for a, b, and c are 1, 2, and 2, respectively.
- 30 is the only multiple of 5 and is not a multiple of 25. 5 must then divide both a and b, and there are no such restrictions for c. The minimum exponent of 5 for a, b, and c are 1, 1, and 0, respectively.

Combining this, the minimum value of abc is $2^{3+1+3} \cdot 3^{1+2+2} \cdot 5^{1+1+0} = 2^7 \cdot 3^5 \cdot 5^2 = 777600$.

11. [**8**] A rectangle with area 22 is inscribed in a circle with radius 5. What is the perimeter of the rectangle?

Proposed by William Roe.

Answer: $\boxed{24}$

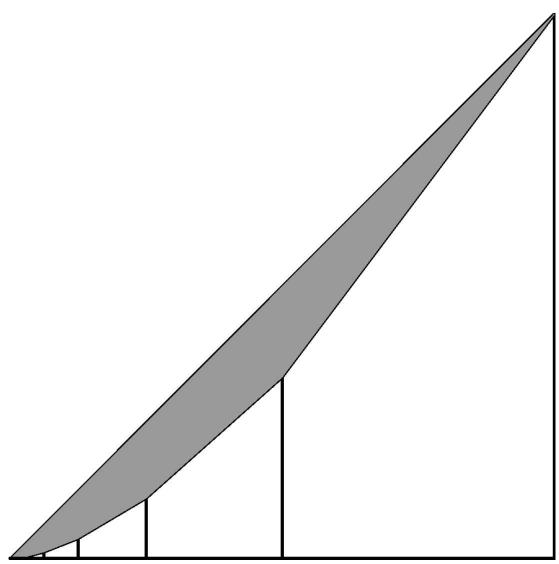
Solution: If a and b are the sides of the rectangle, the hypotenuse must be $2r = 10 = \sqrt{a^2 + b^2}$. From the area, $ab = 22$. The perimeter is $2a + 2b$. Squaring the hypotenuse and adding twice the area gives $a^2 + b^2 + 2ab = 144$. Taking the square root and doubling gives 24 as the perimeter.

12. [**9**] A polygon has infinite vertices, located at $\left(\frac{1}{2^n}, \frac{1}{3^n}\right)$ for all nonnegative integers n. What is the area of the polygon?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{1}{10}}$

Solution:



The area of the polygon can be found by summing the areas of infinitely many trapezoids, then subtracting from the right triangle with vertices at $(0, 0)$, $(1, 1)$, and $(1, 0)$. The area of this right triangle is $\frac{1}{2}$.

The area of the rightmost trapezoid is $\frac{\left(\frac{1}{3}+1\right)\cdot\left(1-\frac{1}{2}\right)}{2} = \frac{1}{3}$. For each subsequent trapezoid, the vertical bases are scaled by a factor of $\frac{1}{3}$ and the horizontal height is scaled by a factor of $\frac{1}{2}$, meaning the area is scaled by a factor of $\frac{1}{6}$. Thus, the sum of all of our trapezoids is the infinite geometric series $\frac{1}{3}+\frac{1}{3}\left(\frac{1}{6}\right)+\frac{1}{3}\left(\frac{1}{6}\right)^2+\cdots$. Using the formula for an infinite geometric series, this sum is $\frac{\frac{1}{3}}{1-\frac{1}{6}}=\frac{2}{5}$. Subtracting from $\frac{1}{2}$ gives the area of the polygon to be $\frac{1}{10}$.

13. [**9**] p and q are chosen at random from the set of all positive integers. What is the probability that, when the fraction $\frac{p}{q}$ is fully simplified, the numerator is even?

Proposed by Lewis Lau.

Answer: $\boxed{\frac{1}{3}}$

Solution: Rewrite $p = a \cdot 2^b$ and $q = c \cdot 2^d$ where a and c are odd. The answer is then the probability $b > d$.

p is odd with $\frac{1}{2}$ chance, so $b = 0$ with $\frac{1}{2}$ chance. p is even with $\frac{1}{2}$ chance, but in $\frac{1}{2}$ of those occurences 4 also divides p. As such, $b = 1$ with $\frac{1}{4}$ chance. More generally, for any choice of k, 2^k divides p with $\frac{1}{2^k}$ chance, but $\frac{1}{2}$ of the time, 2^{k+1} also divides p. As such, $b = k$ with $\frac{1}{2^{k+1}}$ chance. The results are the exact same for d.

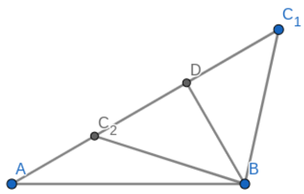
With the distributions of b and d, the chance they are the same is $\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{4}+\cdots=\frac{1}{3}$, and the chance they are different is $\frac{2}{3}$. By symmetry, the chance $b > d$ rather than $b < d$ is half that, $\frac{1}{3}$.

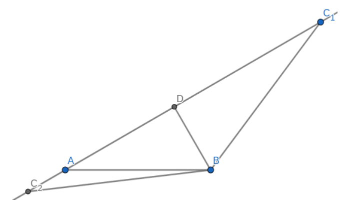
14. [**10**] Olivia has a triangle ABC, and Ivy is trying to guess its area. Olivia tells Ivy that angle A is $30°$ and that side AB equals 10, but Ivy cannot determine the area of ABC with that information alone. Olivia then tells Ivy the value of side BC, and Ivy is able to uniquely determine the triangle's area. What is the sum of all possible positive integers that CANNOT have been the value of BC?

Proposed by Daniel Zhu.

Answer: $\boxed{40}$

Solution: Since Ivy can uniquely determine the triangle's area, she can uniquely determine the triangle. Consider the following two diagrams.





In the first diagram, there are two possible locations for point C, which are reflections of each other other across the perpendicular from B. This occurs because $BC < AB$. In the second diagram, $BC > AB$, so only one of the two possible locations for C actually works; the other is on the wrong side of A.

If C is exactly the foot of the perpendicular from A (point D in the diagrams), there is also only one possible location. Since $\triangle ABD$ is a $30-60-90$ triangle, we know $AD = \frac{1}{2} \cdot 10 = 5$.

Lastly, if $BC < BD$, we get a degenerate triangle, which doesn't work.

Therefore, the values of BC that don't work are when $5 < BC < 10$ and when $5 > BC$. The answer is $1+2+3+4+6+7+8+9 = 40$.

15. [**10**] Define $f(n)$ as the number of divisors of n and $g(n)$ as

$$g(n) := f(n) + \sum_{i=1}^{k-1} g\left(a_i\right)$$

where $(a_1, a_2, \ldots, a_k)$ are the divisors of n in increasing order. Given that $g(1) = 0$, what is $g(72)$?

Proposed by Evan Zhang.

Answer: $\boxed{227}$

Solution: Consider the factors of 72:

1	2	4	8
3	6	12	24
9	18	36	72

The corresponding $f(n)$ are given as follows:

1	2	3	4
2	4	6	8
3	6	9	12

We can create a similar array for $g(n)$. We are given $g(1) = 0$, and each $g(n)$ is the sum of $f(n)$ and all $g(x)$ in the rectangular region above and to the left of n in the $g(n)$ array.

This gives the following $g(n)$ values:

$$\begin{array}{cccc} 0 & 2 & 5 & 11 \\ 2 & 8 & 23 & 59 \\ 5 & 23 & 77 & 227 \end{array}$$

for a final answer of 227.