MBMT Team Round — Weierstrass

March 9, 2025

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

MBMT Team Round Answer Sheet — Weierstrass

March 9, 2025

Team Name ____

Team Number _________

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1. **[4]** Mr. Schwartz has 96 pringles and 120 pieces of candy. What is the largest number of students for which both pringles and candy can be split equally among them?

2. **[4]** It takes Gloria the Snail 40 hours to crawl around a rectangular basketball court and 46 hours to crawl around a rectangular tennis court, which has a perimeter 4 meters longer than the basketball court. If Gloria the Snail crawls at a constant speed, what is Gloria the Snail's speed in meters per hour?

3. **[4]** Let $a \star b = \frac{a+b}{a}$. What is $7 \star (8 \star 7) - 8 \star (7 \star 8)$?

4. **[5]** Kite $ABCD$ is inscribed in a circle. If the area of the kite is 48 square units and BD is 6 units long, what is the area of the circle?

5. **[5]** Valerie draws a right triangle with legs of length 1 and 8. Michelle draws a different right triangle with legs of integer length. To their surprise, the hypotenuses of both right triangles are the same length! What is the area of Michelle's right triangle?

6. **[5]** If $1^3 + 2^3 + 3^3 + \cdots + n^3 = 2025$, what is n?

7. **[6]** Olivia thinks that two plus two equals five. As in, she believes there are solutions to the following equation:

$$\begin{array}{cr} & \text{T W O} \\ + & \text{T W O} \\ \hline & \text{F I V E} \end{array}$$

In Olivia's equation, each letter represents a distinct digit. What is the maximum possible value of $FIVE$?

8. **[6]** Two ants start on the same vertex of a regular hexagon with side length 2 and begin running in opposite directions along the sides of the hexagon. If one ant runs 3 times as fast as the other, what is the distance from the point where they first meet to their starting location?

9. [7] What is the maximum number of intersection points between 3 ellipses and 3 lines?

10. [8] If positive integers a, b, and c satisfy $\gcd(a, b) = 30$, $\gcd(b, c) = 18$, and $\gcd(c, a) = 24$, what is the minimum value of abc?

11. [8] A rectangle with area 22 is inscribed in a circle with radius 5. What is the perimeter of the rectangle?

12. [9] A polygon has infinite vertices, located at $\left(\frac{1}{2^n}, \frac{1}{3^n}\right)$ for all nonnegative integers n. What is the area of the polygon?

13. [9] p and q are chosen at random from the set of all positive integers. What is the probability that, when the fraction $\frac{p}{q}$ is fully simplified, the numerator is even?

14. [10] Olivia has a triangle ABC, and Ivy is trying to guess its area. Olivia tells Ivy that angle A is 30° and that side AB equals 10, but Ivy cannot determine the area of ABC with that information alone. Olivia then tells Ivy the value of side BC, and Ivy is able to uniquely determine the triangle's area. What is the sum of all possible positive integers that CANNOT have been the value of BC?

15. [10] Define $f(n)$ as the number of divisors of n and $g(n)$ as

$$g(n) := f(n) + \sum_{i=1}^{k-1} g(a_i)$$

where $(a_1, a_2, \ldots, a_k)$ are the divisors of n in increasing order. Given that $g(1) = 0$, what is $g(72)$?