1. What is the second-smallest positive integer that is a multiple of both 4 and 6?

Proposed by Lewis Lau.

Answer: $\boxed{24}$

Solution: The smallest positive integer that is divisible by both 4 and 6 is the LCM, which is 12. The next smallest is $2 \cdot 12 = 24$.

2. Stephen correctly computes the product of the first four prime numbers. Ricky also attempts to compute the product of the first four prime numbers, but he mistakenly believes that the first prime number is 1, not 2. What is the positive difference between Stephen's and Ricky's calculations?

Proposed by Jason Youm.

Answer: $\boxed{180}$

Solution: The first four prime numbers (numbers that are only divisible by 1 and themselves) are 2, 3, 5, and 7. Stephen calculates a value of $2 \cdot 3 \cdot 5 \cdot 7 = 210$. Ricky calculates a value of $1 \cdot 2 \cdot 3 \cdot 5 = 30$. The positive difference is $210 - 30 = 180$.

3. Alice's answer to her math homework has been eaten by her pet ants, who only eat their favorite digit. Her answer is now $7X91X8$ where X is a missing digit. If Alice remembers that her answer was divisible by 12, what digit did the ants eat?

Proposed by Ashley Zhang.

Answer: $\boxed{4}$

Solution: Alice's answer must be divisible by 3. Summing up all the digits of her answer gives $25 + 2X$, which must divide 3. The only digits X for which this works are 1, 4, and 7. Alice's answer must additionally divide 4, which means $X8$ must divide 4. The only possible value for X that satisfies both conditions is $X = 4$.

4. Shriyan divides his favorite three-digit number by 2, 3, 4, 8, 9, and 11 and gets a remainder of 1 each time. What is Shriyan's favorite three-digit number?

Proposed by Jason Youm.

Answer: $\boxed{793}$

Solution: Let us find the least common multiple of 2, 3, 4, 8, 9, and 11. We need to have 3 factors of 2, 2 factors of 3, and 1 factor of 11. Multiplying these out gives a value of 792, so if Shriyan's number is 793, it will have a remainder of 1 when divided by each of 2, 3, 4, 8, 9, and 11.

5. Three *consecutive* nonzero digits are taken, and the 6 numbers formed by permuting the digits are added. What is the largest integer that must divide the sum?

Proposed by Evan Zhang.

Answer: $\boxed{666}$

Solution: Let the digits be $n-1$, n, and $n+1$ where $2 \leq n \leq 8$. Each digit appears in each location twice, so the sum is $200((n-1)+n+(n+1)) + 20((n-1)+n+(n+1)) + 2((n-1)+n+(n+1))$. This becomes $222 \cdot 3n = 666n$. 666 clearly divides this. Trying both $n = 2$ and $n = 3$ shows 666 is the largest number that will always divide $666n$.

6. Let $\lfloor x \rfloor$ represent the largest integer less than or equal to x. There exists a unique 5-digit positive integer n such that the sum of its digits is 20 and

$$\left\lfloor \frac{n}{10} \right\rfloor + \left\lfloor \frac{n}{100} \right\rfloor + \left\lfloor \frac{n}{1000} \right\rfloor + \left\lfloor \frac{n}{10000} \right\rfloor = 2025$$

What is the product of the digits of n?

Proposed by Chaewoon Kyoung.

Answer: $\boxed{320}$

Solution: Let $n = \underline{a_1a_2a_3a_4a_5} = a_1 \cdot 10^4 + a_2 \cdot 10^3 + a_3 \cdot 10^2 + a_4 \cdot 10^1 + a_5$, where $a_i \in \{1,2,3,4,5,6,7,8,9,0\}$ for $i = 1,2,3,4,5$, and $a_1 \neq 0$. Then,

$$\begin{aligned}
\left\lfloor \frac{n}{10} \right\rfloor &= a_1 \cdot 10^3 + a_2 \cdot 10^2 + a_3 \cdot 10^1 + a_4, \\
\left\lfloor \frac{n}{100} \right\rfloor &= a_1 \cdot 10^2 + a_2 \cdot 10^1 + a_3, \\
\left\lfloor \frac{n}{1000} \right\rfloor &= a_1 \cdot 10^1 + a_2, \text{and} \\
\left\lfloor \frac{n}{10000} \right\rfloor &= a_1.
\end{aligned}$$

The equation then becomes $a_1 \cdot 1111 + a_2 \cdot 111 + a_3 \cdot 11 + a_4 = 2025$. Since $2 \cdot 1111 > 2025, a_1 < 2$ and therefore $a_1 = 1$.

Then, $a_2 \cdot 111 + a_3 \cdot 11 + a_4 = 2025 - 1111 = 914$. The maximum of $a_3 \cdot 11 + a_4$ is $9 \cdot 11 + 9 = 108$, so $a_2 \cdot 111 \geq 914 - 108 = 806$. Therefore, $a_2 >= 8$.

Also, since $9 \cdot 111 > 914$, $a_2 < 9$ and thus $a_2 = 8$. Then, $a_3 \cdot 11 + a_4 = 914 - 888 = 26$. Since the maximum of a_4 is 9, $a_3 \cdot 11 \geq 26 - 9 = 17$, so $a_3 \geq 2$. Also, since $3 \cdot 11 > 26$, $a_3 < 3$ and thus $a_3 = 2$.

Then $a_4 = 26 - 22 = 4$.

Since $a_1 + a_2 + a_3 + a_4 + a_5 = 20$ is given in the problem, $a_5 = 20 - a_1 - a_2 - a_3 - a_4 = 20 - 1 - 8 - 2 - 4 = 5$. Finally, the product of all digits are $a_1 a_2 a_3 a_4 a_5 = 1 \cdot 8 \cdot 2 \cdot 4 \cdot 5 = 320$.

7. Let $m, n > 2025$ be prime numbers such that $m = n + 2$. What is the remainder when mn is divided by 36?

Proposed by William Roe.

Answer: $\boxed{35}$

Solution: There's nothing too special about 2025 besides the fact that primes larger than it definitely don't divide 2 or 3. If we just take some smaller primes that are easier to work with, say 5, we can just assume they behave the same way as larger primes. $5 \cdot 7 = 35$, $11 \cdot 13 = 143$, and $17 \cdot 19 = 323$. We see that all of these leave a remainder of 35 when divided by 36, so that seems like a reasonable answer to assume.

For a more rigorous solution:

Since n and $n + 2$ are both prime and therefore not multiples of 3, $n + 1$ must be a multiple of 3 since any 3 consecutive integers must contain a multiple of 3. Similarly, we can infer that $n + 1$ must be even. Thus, $n + 1$ is a multiple of 6.

Let $n + 1 = 6a$ for some integer a. Then $mn = (6a + 1)(6a - 1) = 36a^2 - 1$. Since $36a^2$ is divisible by 36, $36a^2 - 1$ leaves a remainder of 35 when divided by 36.

8. Define S_n as the sum of all positive integers less than or equal to n that are relatively prime to n (their greatest common factor with n is equal to 1). What is S_{210}?

Proposed by Tane Park.

Answer: $\boxed{5040}$

Solution: Note that if k is relatively prime to n, $n-k$ will also be relatively prime to n. These two numbers clearly add to n, so their average will be $\frac{n}{2}$. Now we just need to compute the number of integers less than n that are relatively prime to n. This is given by Euler's totient formula as $\varphi(210) = 210\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right) = 48$. The desired sum is therefore $105 \cdot 48 = 5040$.