1. **[3]** What is $2025^{2^{0^{2^5}}}$?

Proposed by Olivia Guo.

Answer: $\boxed{2025}$

Solution: 0 to any non-zero power is always 0, and any non-zero number raised to the power of 0 is always 1. Therefore, this simplifies to $2025^1 = 2025$.

2. **[3]** Bob eats 5 bananas for every 2 bananas Kevin eats. If Bob and Kevin eat 42 bananas together, how many more bananas did Bob eat than Kevin?

Proposed by Ashley Zhang.

Answer: $\boxed{18}$

Solution: For every group of 7 bananas, Bob eats 3 more bananas than Kevin eats. Bob and Kevin together eat 6 groups of 7 bananas each, so Bob eats $6 \cdot 3 = 18$ more bananas than Kevin does.

3. **[3]** Which non-zero digit x satisfies the property that the three-digit number $\overline{x01}$ is divisible by 7?

Proposed by Michelle Gao.

Answer: $\boxed{3}$

Solution: The number $\overline{x01}$ can be written as $100x + 1$. We know $98x$ is divisible by 7 and $100x + 1$ is divisible by 7, so $100x + 1 - 98x = 2x + 1$ must be a multiple of 7. We can then test multiples of 7: $2x + 1 = 0$ has no positive integer solutions, but $2x + 1 = 7$ gives us $x = 3$. Therefore $x = 3$ is our solution.

4. **[3]** Two trucks, one of length 7 feet and the other of length 11 feet, are passing each other in opposite directions. If the first truck is driving at 6 feet per second and 2 seconds elapse from when the trucks begin passing each other to when they are fully past each other, how fast is the second truck traveling, in feet per second?

Proposed by Kele Zhang.

Answer: $\boxed{3}$

Solution: When the front of the first truck reaches the back of the second truck, the two trucks have together traveled the length of the second truck. (Same as if the second truck were staying still.) When the back of the first truck reaches the back of the second truck, the two trucks have together traveled an additional length of the first truck. Therefore, the two trucks together travel a total of $7 + 11 = 18$ feet.

In 2 seconds, the first truck travels $2 \cdot 6 = 12$ feet, so the second truck must travel $18 - 12 = 6$ feet. Therefore, the speed of the second truck is $6/2 = 3$ feet per second.

5. **[3]** Arjun is on an elevator starting on the 1st floor. If the elevator moves up at a constant speed, what is the ratio of the time it takes him to go to the 25th floor to the time it takes him to go to the 5th floor?

Proposed by Evan Zhang.

Answer: $\boxed{6}$

Solution: To get to the 25th floor, Arjun goes up $25 - 1 = 24$ floors. To get the to 5th floor, Arjun goes up $5 - 1 = 4$ floors. Going to the 25th floor is $\frac{24}{4} = 6$ times the distance as going to the 5th floor, so it will take 6 times the amount of time.

6. **[4]** What is the maximum number of intersection points between two squares?

Proposed by Olivia Guo.

Answer: $\boxed{8}$

Solution: Suppose the first square is fixed in space. For each side of the second square, it can either not intersect the first square at all, intersect exactly at a vertex, or cut across a vertex intersecting two different sides. Therefore, each side can intersect the square at most twice, so the 4 sides all together can intersect the square at most 8 times.

7. **[4]** x and y are real numbers. Gloria the Cicada wants to estimate the value of $x - y$, so she rounds x down by 0.05 and y up by 0.02. What is the positive difference between Gloria the Cicada's estimated value and her actual value?

Proposed by Reanna Jin.

Answer: $\boxed{0.07}$

Solution: By rounding x down by 0.05, Gloria the Cicada is subtracting 0.05 from the actual value of $x - y$. By rounding y up by 0.02, Gloria the Cicada is subtracting 0.02 more than she should. Therefore, the total differences is $0.05 + 0.02 = 0.07$.

8. **[4]** Jason draws a rocket. Sadly, he is very bad at drawing, so his rocket simply consists of an isosceles triangle sharing its base with one side of a rectangle. Given that each side of the rocket is an integer, if the rectangle has perimeter 40 and the isosceles triangle has perimeter 24, what is the largest possible perimeter of Jason's rocket?

Proposed by Yunyi Ling.

Answer: $\boxed{60}$

Solution: Let the length of the side that the triangle and rectangle share be l. Then, the total perimeter of Jason's rocket is $40 + 24 - 2l = 64 - 2l$. So, we want to minimize l.

Since every side of the rocket is an integer, $\frac{24-l}{2}$ is an integer, so the smallest possible value of l is 2, and we get te largest possible perimeter is $64 - 2 \cdot 2 = 60$.

9. **[4]** Kian is buying pizza. A pizza with a diameter of 5 inches costs him 100 dollars. How much should a pizza with the same thickness and a diameter of 12 inches cost to have the same amount of pizza per dollar?

Proposed by Evan Zhang.

Answer: $\boxed{576}$

Solution: A pizza with diameter 5 inches has area 25π square inches, and costs 100 dollars. A pizza with diameter 12 inches has area 144π square inches, so it should cost $\frac{144}{25} \cdot 100 = 144 \cdot 4 = 576$ dollars.

10. **[4]** Evan was biking to RTC, but $\frac{2}{3}$ of the way there, his bike broke down, so he walked the rest of the way there. If he walked for $\frac{2}{3}$ of the total time, how many times faster is his biking speed than his walking speed?

Proposed by Ivy Guo.

Answer: $\boxed{4}$

Solution: We can without loss of generality assume his walking speed is 1, his biking speed is b, and the total distance is d. Then, the amount of time he spent biking is $\frac{\frac{2}{3}d}{b}$ and the amount of time he spent walking is $\frac{\frac{1}{3}d}{1}$.

Since he spent twice as much time walking than biking, we have

$$\frac{1}{3}d = 2 \cdot \frac{2}{3b}d.$$

Solving, we get $bd = 4d$, so $b = 4$.

11. **[5]** A hexagon is inscribed in a circle. Yunyi draws two triangles, using each vertex of the hexagon exactly once. What is the probability that the two triangles intersect?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{7}{10}}$

Solution: Let our hexagon be $ABCDEF$.

There are $\frac{1}{2}\binom{6}{3}\binom{3}{3} = 10$ ways to draw two triangles from the hexagon.

For the two triangles to not intersect, both triangles must be made up of three adjacent vertices. WLOG, let the first triangle have vertex A. Then, there are three possible triangles to draw: ABC, FAB, and EFA. In each case, the second triangle is determined. So, our answer is $1 - \frac{3}{10} = \frac{7}{10}$.

12. **[5]** Peter and Caleb are trading business cards. Caleb has triple Peter's number of cards. Peter then gives Caleb 3 cards, and Caleb now has quadruple Peter's number of cards. How many cards do they have in total?

Proposed by Evan Zhang.

Answer: $\boxed{60}$

Solution: Let Peter have p cards and Caleb have c cards. Before Peter gives Caleb any cards, we have $c = 3p$. Afterwards, Peter has $p - 3$ cards and Caleb has $c + 3$ cards, so $c + 3 = 4(p - 3)$. So, $3p + 3 = 4p - 12$, so $p = 15$ and $c = 45$. So, they have 60 cards total.

13. **[5]** Jimmy's least favorite number is between 0 and 64. He decides to add up all the numbers from 0 to 64, leaving out his least favorite number. If he correctly finds the sum to be 2025, what is Jimmy's least favorite number?

Proposed by William Roe.

Answer: $\boxed{55}$

Solution: The sum of all numbers from 0 to 64 is $\frac{64 \cdot 65}{2} = 2080$. Jimmy left out his favorite number and got a sum of 2025, meaning his favorite number is $2080 - 2025 = 55$.

14. **[5]** If $x^2 = (7 \cdot 9 + 1)(13 \cdot 15 + 1)(19 \cdot 21 + 1)$, what is the value of x?

Proposed by Michelle Gao.

Answer: $\boxed{2240}$

Solution: $7 \cdot 9 + 1 = (8 - 1)(8 + 1) + 1 = 8^2 - 1^1 + 1 = 8^2$. For the same reason, $13 \cdot 15 + 1 = 14^2$ and $19 \cdot 21 + 1 = 20^2$.

So, $x^2 = 8^2 \cdot 14^2 \cdot 20^2$, and $x = 8 \cdot 14 \cdot 20 = 2240$.

15. **[5]** Yunyi has a cube and notices that its volume and surface area have the same numerical value a. Kian has a sphere and also notices that its volume and surface area have the same numerical value b. What is $\frac{a}{b}$?

Proposed by Evan Zhang.

Answer: $\boxed{\dfrac{6}{\pi}}$

Solution: If the cube has side length s, its volume is s^3 and its surface area is $6s^2$. Setting them equal, $s = 6$ and $a = 216$.

If the sphere has radius r, its volume is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$. Seting them equal, $r = 3$ and $b = 36\pi$.

Dividing gives $\frac{a}{b} = \frac{6}{\pi}$.

16. **[7]** A frog is at the point $(0,0)$ on the coordinate plane and spots a fly at the point $(4,2)$. At each step, if the frog is at the point (x,y), it can move to the points $(x+1,y)$, $(x,y+1)$, or $(x+1,y+1)$. How many sequences of moves are there for the frog to reach the fly?

Proposed by Tony Song.

Answer: $\boxed{41}$

Solution: Let $f(x,y)$ be the number of ways for the frog to get to point (x,y). The frog can either come from $(x-1,y)$, $(x,y-1)$, or $(x-1,y-1)$. So, $f(x,y) = f(x-1,y) + f(x,y-1) + f(x-1,y-1)$. We can then draw the following diagram, where the label on a point is equal to the number of ways to get to that point, to see the answer is 41.

$$\begin{array}{ccccccccc} 1 & — & 5 & — & 13 & — & 25 & — & 41 \\ | & / & | & / & | & / & | & / & | \\ 1 & — & 3 & — & 5 & — & 7 & — & 9 \\ | & / & | & / & | & / & | & / & | \\ 1 & — & 1 & — & 1 & — & 1 & — & 1 \end{array}$$

17. **[7]** Leroy the Leg Lover lords over his legendary legion of leggy pets. Soggy squids have 10 legs each and outstanding octopi have 8 legs each. If Leroy has more squids than octopi, and his leggy total tallies to 134, how many animals does he have?

Proposed by Tony Song.

Answer: $\boxed{14}$

Solution: Let x be the number of squids and y be the number of octopi. The conditions are $10x + 8y = 134$ and $x > y$. With the inequality, $18x > 10x + 8y = 134$, so $x > \frac{134}{18} \approx 7.4$ and x is at least 8. Rearranging the first to $2x + 8(x+y) = 134$. 8 must divide $134 - 2x$, so x is:

$$\begin{aligned} 134 &\equiv 2x \pmod 8 \\ 6 &\equiv 2x \pmod 8 \\ 3 &\equiv x \pmod 4). \end{aligned}$$

The smallest such x is 11, which we can see works with $y = 3$. As such, the answer is $x + y = 14$.

18. [**7**] A circle with unknown radius passes through vertices A and B of rectangle $ABCD$. The circle is tangent to side DC and intersects side BC at E so that $BE = 10$ and $CE = 8$. What is the length of side AB?

Proposed by Ruixi Zhang.

Answer: $\boxed{24}$

Solution: Let M be the midpoint of side DC. By symmetry, the circle must pass through point M. Then, by Power of a Point, we have $CE \cdot CB = CM^2$, so $CM^2 = 8 \cdot 18 = 144$. Therefore, $CM = 12$, so $AB = DC = 2CM = 24$.

19. [**7**] What is the 100th smallest positive integer that contains only odd digits?

Proposed by Evan Zhang.

Answer: $\boxed{579}$

Solution: There are 5 odd digits, so there are 5 1-digit numbers that contain only odd digits, $5 \cdot 5 = 25$ 2-digit numbers that contain only odd digits, and $5 \cdot 5 \cdot 5 = 125$ 3-digit numbers contain only odd digits. Therefore the 100th smallest number that contains only odd digits must be a 3-digit number. If the 3-digit number starts with either 1, 3, or 5 there are 3 choices for the first digit, and 5 for each the second and third digits, for a total of $3 \cdot 5 \cdot 5 = 75$ numbers. Since we already have 30 numbers with 1 or 2 digits, the 105th number that contains only odd digits must be 599. Counting backwards, we get that the 100th number that contains only odd digits is 579.

20. [**7**] Triangle ABC has $AB = 6$, $BC = 8$, and $AC = 10$. If Alice chooses a random point within the triangle, what is the probability that the point lies in the incircle of ABC?

Proposed by Andy Yan.

Answer: $\boxed{\dfrac{\pi}{6}}$

Solution: Since $6^2 + 8^2 = 10^2$, triangle ABC is a right triangle. Let the radius of the incircle be r. Then, the tangent distance from B to the incircle must also be r. AC is the sum of the tangent distance from A to the circle and from C to the circle, so $AC = 6 - r + 8 - r$. Therefore, $10 = 14 - 2r$, so $r = 2$. The area of the incircle is therefore 4π, and the area of triangle ABC is $\frac{1}{2} \cdot 6 \cdot 8 = 24$, so the probability the point lies in the incircle is $\frac{4\pi}{24} = \frac{\pi}{6}$.

21. [**9**] For a positive integer n, let $f(n)$ represent the largest prime number that divides $n!$. What is the sum of $f(n)$ as n goes from 2 to 13, inclusive?

Proposed by Lewis Lau.

Answer: $\boxed{81}$

Solution: Any prime less than or equal to n will divide $n!$. Any prime larger than n will not. Therefore, $f(n)$ is the largest prime number less than n. Adding these up gives $2+3+3+5+5+7+7+7+7+11+11+13 = 81$.

22. [**9**] Stephen rolls a fair six sided die. He then randomly paints a number of faces of a cube corresponding to the number shown on the die (if he rolls a 4, he paints 4 random faces). Stephen then cuts the cube into 64 smaller cubes and picks one at random to roll. What is the probability that the top face of the small cube is painted?

Proposed by Evan Zhang.

Answer: $\boxed{\dfrac{7}{48}}$

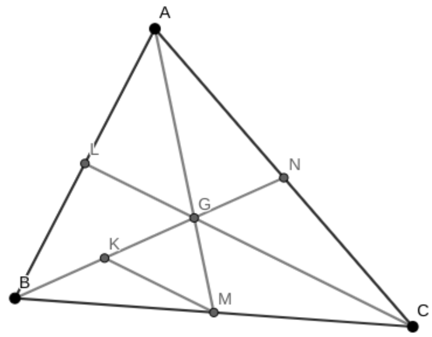
Solution: The probability that the top face is painted is $\frac{\text{painted faces}}{\text{faces}}$. The denominator is always $6 \cdot 64 = 384$. If Stephen initially rolls n, the numerator is $16n$. The expected value of n is $\frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2}$, so the average number of painted faces is $16 \cdot \frac{7}{2} = 56$. Dividing, the answer is $\frac{56}{384} = \frac{7}{48}$.

23. [**9**] Ivy has a triangle with sides of length 13, 14, and 15. Olivia comes and makes another triangle whose side lengths are the lengths of the medians of Ivy's triangle. What is the area of Olivia's triangle?

Proposed by Chaewoon Kyoung.

Answer: $\boxed{63}$

Solution:



Let AM, BN, CL the 3 medians of the triangle. They intersect on one point, G, which is the centroid of the $\triangle ABC$. Let K be the midpoint of BG. Then, $KG = \frac{1}{2}BG$. Also, by midpoint theorem, $KM = \frac{1}{2}GC$. Since G divides the 3 medians with ratio $2:1$, the 3 sides of $\triangle GKM$ are $\frac{1}{3}$ of each median. Therefore, by SSS similarity, the area of the triangle made with 3 medians of $\triangle ABC$ is $9[KGM]$. Also, $[GKM] = \frac{1}{2}[GBM] = \frac{1}{2}\left(\frac{1}{2}[GBC]\right) = \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{3}[ABC]\right)\right) = \frac{1}{12}[ABC]$.

To find the area of $\triangle ABC$, we find the altitude to side BC. If the altitude has length x, the other legs of the right triangles are $\sqrt{169-x^2}$ and $\sqrt{225-x^2}$. They must add up to 14, so we have

$$\begin{aligned}
\sqrt{169-x^2}+\sqrt{225-x^2} &= 14\\
\sqrt{225-x^2} &= 14-\sqrt{169-x^2}\\
225-x^2 &= 196+169-x^2-28\sqrt{169-x^2}\\
28\sqrt{169-x^2} &= 140\\
\sqrt{169-x^2} &= 5\\
169-x^2 &= 25\\
x &= 12.
\end{aligned}$$

Therefore, $[KGM] = \frac{1}{12}\cdot 84 = 7$, and the answer is $9\cdot 7 = 63$.

Another way to find the area is using Heron's formula. We have $s = \frac{1}{2}(13+14+15) = 21$, so

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21\cdot 8\cdot 7\cdot 6} = 84.$$

24. **[9]** Alex picks two distinct primes p_1 and p_2, then makes a list of all positive integers greater than 1 whose prime factorizations contain no primes other than p_1 and p_2. He then takes the reciprocal of every number on the list, and adds them together. If he gets a simplified fraction with a denominator of 2024, what is the numerator of his fraction?

Proposed by Evan Zhang.

Answer: $\boxed{1015}$

Solution: Any number whose prime factorization contains only primes p_1 and p_2 can be written as $p_1^{e_1}p_2^{e_2}$ so its reciprocal is $\frac{1}{p_1^{e_1}p_2^{e_2}}$. The sum of this value over all nonnegative integers e_1 and e_2 is given as

$$\left(1+\frac{1}{p_1}+\frac{1}{p_1^2}+\cdots\right)\left(1+\frac{1}{p_2}+\frac{1}{p_2^2}+\cdots\right).$$

Which can be simplified via the infinite geometric sum into

$$\left(\frac{1}{1-\frac{1}{p_1}}\right)\left(\frac{1}{1-\frac{1}{p_2}}\right)=\frac{p_1p_1}{(1-p_1)(1-p_2)}.$$

To get the denominator to be 2024, we must have $p_1 = 3$ and $p_2 = 1013$. This gives $p_1p_2 = 3039$. However, as we are looking at the positive integers greater than 1, we must subtract 1, giving us a final fraction of $\frac{3039}{2024} - 1 = \frac{1015}{2024}$ for an answer of 1015.

25. **[9]** For a given value k, let (x, y) be a random point such that $-10 \le x, y \le 10$ and $x + y = k$. Let $f(k)$ be the expected value of $|x| + |y|$. If k can be any real number, what is the minimum possible value of $f(k)$?

Proposed by Ivy Guo.

Answer: $\boxed{20\sqrt{2} - 20}$

Solution: If k is greater than 10, both x and y must be positive, so the expected value is just k.

Assume $k \le 10$. We can also assume $k > 0$, because the $k < 0$ case is symmetric. When $0 \le x \le k$, both x and y are positive, so the expected value is k. When $k \le x \le 10$, the average expected value is $\frac{k+10}{2} + \frac{10-k}{2} = 10$. When $k-10 \le x \le 0$, the average expected value is also 10 by symmetry.

Therefore,

$$f(x) = \frac{k}{20-k}\cdot k + \frac{20-2k}{20-k}\cdot 10 = -k + \frac{200}{20-k}.$$

Let $S = -k + \frac{200}{20-k}$. By the AM-GM inequality,

$$20 + S = 20 - k + \frac{200}{20-k} \geq 2\sqrt{200},$$

so $S \geq 20\sqrt{2} - 20$. This is less than 10, so it's minimal.

26. **[12]** Let $f(x)$ be the product

$$\left(1 + \frac{1}{10^6}\right)\left(1 + \frac{2}{10^6}\right)\cdots\left(1 + \frac{x}{10^6}\right).$$

Estimate the smallest positive integer x such that $f(x) > 10^6$.

Proposed by Evan Zhang.

Answer: $\boxed{5261}$

Solution: Let's first work under the assumption that x is fairly small compared to 10^6. The binomial approximation tells us that for small x, $(1+x)^n \approx 1 + nx$. Then we can approximate our given expression as

$$\left(1 + \frac{1}{10^6}\right)\left(1 + \frac{1}{10^6}\right)^2\cdots\left(1 + \frac{1}{10^6}\right)^x = \left(1 + \frac{1}{10^6}\right)^{\frac{x(x+1)}{2}}.$$

We can use the approximation that $\left(1 + \frac{1}{x}\right)^x \approx e$ for large x, and since 10^6 is fairly large, we can approximate $\left(1 + \frac{1}{10^6}\right)^{10^6} \approx e$. Our expression can now be approximated as

$$\left(1 + \frac{1}{10^6}\right)^{\frac{x(x+1)}{2}} = \left(1 + \frac{1}{10^6}\right)^{10^6 \cdot \frac{x(x+1)}{2\cdot 10^6}} \approx e^{\frac{x(x+1)}{2\cdot 10^6}}.$$

Let's try to estimate $e^n = 10^6$. We know $e \approx 2.71$, so $e^2 \approx 7.5$, $e^4 \approx 56$, and $e^7 \approx 1100$. This gives us, loosely estimating, that $e^{14} \approx 10^6$. Going back, this gives us $\frac{x(x+1)}{2\cdot 10^6} \approx 14$.

Since x is large, $x(x+1) \approx x^2$, so our estimate becomes $x^2 \approx 28 \cdot 10^6$ and $x \approx 5300$.

27. **[12]** Square $A_0B_0C_0D_0$ has a side length of 1. For all positive integers n, square $A_nB_nC_nD_n$ is formed by inscribing an equilateral triangle in square $A_{n-1}B_{n-1}C_{n-1}D_{n-1}$ such that a vertex of the triangle coincides with a vertex of the square, and then inscribing a square in the equilateral triangle. Find

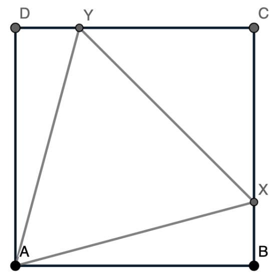
$$\sum_{i=0}^{\infty}[A_iB_iC_iD_i]$$

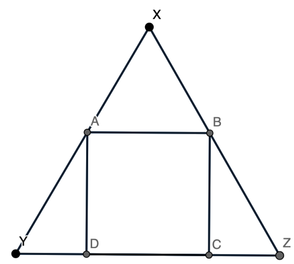
where $[A_iB_iC_iD_i]$ denotes the area of square $A_iB_iC_iD_i$.

Proposed by Evan Zhang.

Answer: $\boxed{1.30014435}$

Solution: We can break the problem down into two parts: a triangle inscribed in a square, and a square inscribed inside the triangle. The sum of the areas of the squares will be an infinite geometric series with first term 1. We just need to find the common ratio, which is given by $\frac{[A_{i+1}B_{i+1}C_{i+1}D_{i+1}]}{[A_iB_iC_iD_i]}$.





Consider equilateral triangle AXY inscribed in square $ABCD$ with side length 1 such that X lies on side BC and Y lies on side CD. Call $XC = YC = x$. Then, $YD = 1 - x$. We can apply the Pythagorean theorem to the sides of the equilateral triangle to get

$$XY = AY \implies x^2 + x^2 = 1 + (1-x)^2 \implies x^2 - 2x + 2 = 0.$$

Applying the quadratic formula gives $x = \sqrt{3} - 1$, so the equilateral triangle has sides of length $\sqrt{6} - \sqrt{2}$.

Now, consider square $ABCD$ inscribed in equilateral triangle XYZ with side length 1 such that A lies on side XY and B lies on side XZ. The altitude from X intersects sides AB and CD at X_1 and X_2, respectively. Call $AB = BC = CD = DA = x$. We have $AX_1 = \frac{x}{2}$ so $XX_1 = \frac{x\sqrt{3}}{2}$. Adding up the vertical lengths gives

$$XX_1 + DA = XX_2 \implies \frac{x\sqrt{3}}{2} + x = \frac{\sqrt{3}}{2}$$
$$\implies \left(2 + \sqrt{3}\right)x = \sqrt{3} \implies x = 2\sqrt{3} - 3.$$

The ratio of the lengths $\frac{A_{i+1}B_{i+1}}{A_iB_i} = \left(\sqrt{6} - \sqrt{2}\right)\left(2\sqrt{3} - 3\right) = 9\sqrt{2} - 5\sqrt{6}$. Thus, the ratio of the areas $\frac{[A_{i+1}B_{i+1}C_{i+1}D_{i+1}]}{[A_iB_iC_iD_i]} = \left(9\sqrt{2} - 5\sqrt{6}\right)^2 = 312 - 180\sqrt{3}$. The sum of an infinite geometric sequence with first term a and common ratio r is given by $\frac{a}{1-r}$, so we can evaluate:

$$\sum_{i=0}^{\infty}[A_iB_iC_iD_i] = \frac{1}{1 - \left(312 - 180\sqrt{3}\right)} = \frac{180\sqrt{3} + 311}{479} \approx 1.30014435.$$

A simpler way to get a good estimate is by approximating $\frac{A_{i+1}B_{i+1}}{A_iB_i}$. By using $\sqrt{2} \approx 1.414$ and $\sqrt{6} \approx 2.45$, we get $9\sqrt{2} - 5\sqrt{6} \approx 0.476$. We therefore want $\frac{1}{1-0.476^2}$. 0.476 is slightly less than $\frac{1}{2}$, so our final answer will be slightly less than $\frac{4}{3} = 1.333$.

28. [**12**] 3 random integers a, b, and c are chosen with replacement from -2025 to 2025 inclusive. Estimate the probability that the quadratic $ax^2 + bx + c$ has at least 1 real root.

Proposed by Evan Zhang.

Answer: $\boxed{0.6272126878}$

Solution: We want $ax^2 + bx + c$ to have at least one real root, so we're looking for the probability that $b^2 - 4ac \geq 0$, or $b^2 \geq 4ac$. Notice that if a and c are opposite signs, the expression will always be positive. This happens $\frac{1}{2}$ of the time.

Because there is a factor of 4 on the right side, we can look at the approximately $\frac{1}{4}$ of cases in which $|a|$ and $|c|$ are both less than $\frac{2025}{2}$. In these cases, we can assume the inequality holds $\frac{1}{2}$ of the time, giving us an additional $\frac{1}{8}$. In the other cases, where at least one of $|a|$ and $|c|$ is greater than $\frac{2025}{2}$, we can assume that the inequality rarely holds, giving us an estimate of $\frac{1}{2} + \frac{1}{8} = \frac{5}{8} = 0.625$.

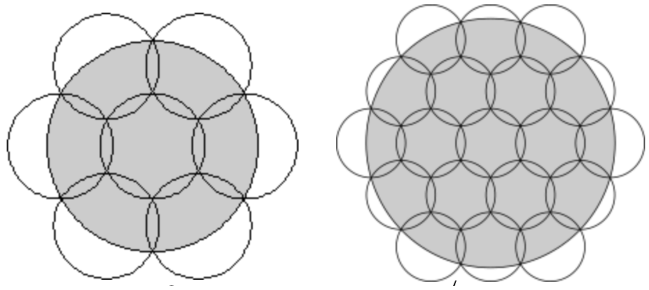
29. [**12**] Yunyi has a circular table with a radius of 2025 inches. He also has an infinite number of stickers in the shape of a circle with a radius of 1 inch. If stickers can overlap and hang off the edge of the table, what is the minimum number of stickers Yunyi needs to cover the entirety of the circle's surface with stickers?

Proposed by Evan Zhang.

Answer: $\boxed{\approx 5468850}$

Solution: The disk covering problem is simplest when working with layers of hexagons, and is otherwise near impossible for large numbers, so this solution will extrapolate off those ideas (see the diagrams below).

We can start off by placing a unit circle directly in the middle of the larger cicle. A layer of circles is defined by subsequent hexagonal rings of circles around the middle. The first layer has just one circle, the next has 6, and each subsequent layer has 6 more circles than the previous one. If we have n layers of circles, we therefore have a total of $1 + 6 \cdot \frac{n(n-1)}{2} = 1 + 3n(n-1)$ circles.



We want the centers of the outermost layer of circles to lie on the circumference of the larger circle. We can then break the problem down into two cases based on the parity of n. If n is even (left diagram), it is equal to $2k$ for some integer k. The radius of the large circle is $3k-1$. Otherwise, if n is odd (right diagram), it is equal to $2k+1$ for some integer k. The radius of the large circle is then $\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(3k+\frac{1}{2}\right)^2}$.

When $k=675$ the $n=2k=1350$ case gives a larger circle radius of 2024. Meanwhile, the $n=2k+1=1351$ case gives a larger circle radius of approximately 2025.5. To get an approximation, we can take $n=1350+\frac{2}{3}=\frac{4052}{3}$ so the radius is approximately 2025. Plugging this in to our previously established formula, we get a total of $1+3\left(\frac{4052}{3}\right)\left(\frac{4049}{3}\right)\approx 5468850$

30. **[12]** Kian is in a room with 2025 lightbulbs which are all initially off. Every second he picks one of the lightbulbs at random and flips its state (if it was off initially, he turns it on, and vice versa). Estimate the floor of the log of the expected number of seconds it'll take for him to turn all of the lightbulbs on.

Proposed by Evan Zhang.

Answer: $\boxed{609}$

Solution: We can treat this problem as walking along a number line bounded by 0 and 2025, where the position represents the number of lightbulbs that are on. While at point x, a move has a probability of $\frac{x}{2025}$ of moving to point $x-1$, and a $\frac{2025-x}{2025}$ of moving to point $x+1$.

Define $f(x)$ as the expected number of moves required to get to $x+1$ when standing at x. It will take an expected $\frac{1}{\frac{2025-x}{2025}}=\frac{2025}{2025-x}$ tries to move forward one step, so we will move back an expected $1-\frac{2025}{2025-x}=\frac{x}{2025-x}$ times. This gives the following recursive formula:

$$f(x)=\frac{x}{2025-x}(1+f(x-1))+1$$

where $f(0)$ is clearly just 1.

Taking the following sum gives the final answer:

$$\sum_{n=0}^{2024} f(n) \approx 3.85 \times 10^{609} \Rightarrow 609.$$

Curiously, our answer is very close to 2^{2025}. In fact, it approaches 2^x as the number of lightbulbs x gets large. To show this, we can see that once $x-1$ of the lightbulbs are turned on, the last one has a roughly equal chance of either being on or off, so the time it takes to turn on all the lightbulbs approximately doubles with each additional lightbulb. Using the approximation of $\log(2) \approx 0.301$ gives an approximation of $0.301 \cdot 2025 = 609.525$.