1. Three points A, T, and W lie on a circle. A and W are the endpoints of the circle's diameter, and T is equidistant from A and W. What is the measure, in degrees, of $\angle ATW$?

Proposed by Lewis Lau.

Answer: $\boxed{90^\circ}$

Solution: The measure of an inscribed angle is half the angle measure of the subtended arc. Since AW is a diameter, the subtended arc measures 180°, so $\angle ATW = 90^\circ$.

2. $\triangle ABC$ is a right triangle with $AB = 6$, $BC = 8$, and $AC = 10$. A point P is chosen on AC so that both triangles $\triangle ABP$ and $\triangle BCP$ are isosceles. What is the length of BP?

Proposed by Thomas Yao.

Answer: $\boxed{5}$

Solution: The hypotenuse of any right triangle is the diameter of its circumcircle due to the Inscribed Angle Theorem. Thus, the midpoint of AC is the center of the circumcircle and by definition equidistance from points A, B, and C. This means P is the midpoint of AC so that $\triangle ABP$ and $\triangle BCP$ are isosceles with $AP = BP = CP = 5$.

3. 4 points in the plane form the vertices of a convex quadrilateral. There are exactly four unique triangles that can be formed using three of four points. If the sum of the areas of these four triangles is 24, what is the area of the quadrilateral?

Proposed by Sam Easaw.

Answer: $\boxed{12}$

Solution: If we draw out the two diagonals, we split the quadrilateral into 4 smaller triangles. Each of the unique triangles in the problem are made up of 2 of these smaller triangles. Adding up the areas of the 4 unique triangles adds each small triangle twice, as each small triangle is contained within two of the unique triangles. The area of the quadrilateral is the area of the small triangles, which is $\frac{24}{2} = 12$.

4. A rectangular prism has dimensions $2 \times 3 \times 4$. An octahedron is formed whose vertices are the centers of the faces of the rectangular prism. What is the volume of the octahedron?

Proposed by Ivy Guo.

Answer: $\boxed{4}$

Solution: Consider cutting the rectangular prism into 8 octants; the piece of the octahedron in each octant is a tetrahedron that shares 3 sides with the sides of the octant.

The volume of a pyramid is $\frac{1}{3}bh$, where b is the area of the base and h is the height. The base is a right triangle with legs 2 and 1, so its area is $\frac{1}{2} \cdot 2 \cdot 1 = 1$. The height is 1.5. Therefore, the volume of the tetrahedron is $\frac{1}{3} \cdot 1.5 = \frac{1}{2}$.

There are 8 of these tetrahedrons, one in each octant, so the total volume is $8 \cdot 0.5 = 4$.

5. A circle of radius 17 is centered at the origin. Another circle is centered at the point $(21, 0)$, and the two circles share a common chord of length 16. What is the radius of the second circle?

Proposed by Evan Zhang.

Answer: $\boxed{10}$

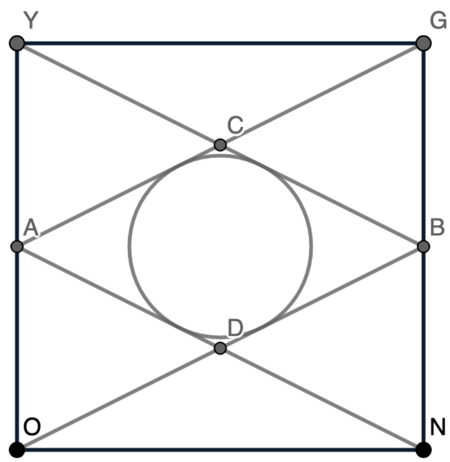
Solution: Let the centers of the two circles be O_1 and O_2, and let the common chord have endpoints A and B. Let P be the point of intersection between AB and O_1O_2. Since $\triangle O_1PA$ is right, we know $O_1P = 15$ by the Pythagorean Theorem. This gives $O_2P = 21 - 15 = 6$. Since O_2PA is also right, by the Pythagorean Theorem again, $O_2A = \sqrt{6^2 + 8^2} = 10$.

6. $YONG$ is a square with side length 10. Let A be the midpoint of YO and B be the midpoint of NG. Let C be the intersection of YB and GA, and D be the intersection of NA and OB. What is the radius of the circle inscribed in rhombus $ABCD$?

Proposed by Evan Zhang.

Answer: $\boxed{\sqrt{5}}$

Solution:



We can solve this by finding the area of the rhombus in two ways. First of all, since $\triangle BCD$ is similar to $\triangle BYO$ in a $1:2$ ratio, $CD = 5$. Since $AB = 10$, the area is $\frac{5\cdot 10}{2} = 25$.

By the Pythagorean Theorem, $AG = \sqrt{5^2 + 10^2} = 5\sqrt{5}$. Thus, $AC = \frac{5\sqrt{5}}{2}$. If the inradius is r, we can split the rhombus into four triangles to get a total area of $2r \cdot AC = 5\sqrt{5} \cdot r$.

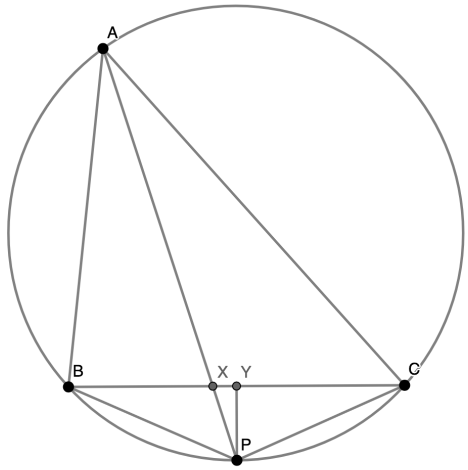
Setting the two areas equal gives $5\sqrt{5} \cdot r = 25$, so $r = \sqrt{5}$.

7. Triangle ABC has $BC = 30$. Point X lies on segment BC such that AX bisects $\angle BAC$. AX is extended past point X and intersects the circumcircle of ABC at another point P different from A. Point Y lies on BC such that PY is perpendicular to BC. If PY has length 8, what is the perimeter of $\triangle BPC$?

Proposed by Yunyi Ling.

Answer: $\boxed{64}$

Solution:



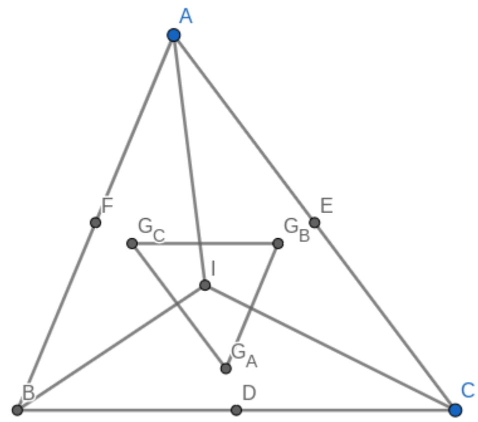
Since $\angle BAP = \angle CAP$, arc BP equals arc CP, which means P bisects arc BC. Thus, $\triangle BPC$ is isosceles with $BP = CP$, meaning $BY = CY$. Since $PY = 8$ and $BY = 15$, by the Pythagorean Theorem $PB = 17$. The perimeter of $\triangle BPC$ is therefore $17 \cdot 2 + 30 = 64$.

8. $\triangle ABC$ is a triangle with $AB = 13$, $BC = 14$, $CA = 15$. Let I be the incenter of ABC, G_A be the centroid of $\triangle IBC$, G_B be the centroid of $\triangle ICA$, and G_C be the centroid of $\triangle IAB$. What is the area of triangle $\triangle G_AG_BG_C$? (The incenter is the intersection point of the three angle bisectors and the centroid is the intersection point of the three medians).

Proposed by Evan Zhang.

Answer: $\boxed{\dfrac{28}{3}}$

Solution:



Let D, E, and F be the midpoints of BC, CA, and AB, respectively. G_A is then the point $\frac{2}{3}$ of the distance from I to D, and similarly for G_B and G_C. Thus, the area of $\triangle G_AG_BG_C$ is $\left(\frac{2}{3}\right)^2$ the area of $\triangle DEF$. As the medial triangle, that is $\frac{1}{4}$ the area of $\triangle ABC$. The area of $\triangle G_AG_BG_C$ is then $\frac{4}{9} \cdot \frac{1}{4} = \frac{1}{9}$ the area of $\triangle ABC$.

To find the area of $\triangle ABC$, we find the altitude to side BC. If the altitude has length x, the other legs of the right triangles are $\sqrt{169 - x^2}$ and $\sqrt{225 - x^2}$. They must add up to 14, so we have

$$\begin{aligned}
\sqrt{169 - x^2} + \sqrt{225 - x^2} &= 14 \\
\sqrt{225 - x^2} &= 14 - \sqrt{169 - x^2} \\
225 - x^2 &= 196 + 169 - x^2 - 28\sqrt{169 - x^2} \\
28\sqrt{169 - x^2} &= 140 \\
\sqrt{169 - x^2} &= 5 \\
169 - x^2 &= 25 \\
x &= 12.
\end{aligned}$$

We find the area to be $\frac{14 \cdot 12}{2} = 84$, so the area of $\triangle G_AG_BG_C$ is $\frac{84}{9} = \frac{28}{3}$.

Another way to find the area is using Heron's formula. We have $s = \frac{1}{2}(13 + 14 + 15) = 21$, so

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84.$$