MBMT Geometry Round — Weierstrass

March 9, 2025

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1.** Three points A, T, and W lie on a circle. A and W are the endpoints of the circle's diameter, and T is equidistant from A and W. What is the measure, in degrees, of $\angle ATW$?

_____ **2.** $\triangle ABC$ is a right triangle with $AB = 6$, $BC = 8$, and $AC = 10$. A point P is chosen on AC so that both triangles $\triangle ABP$ and $\triangle BCP$ are isosceles. What is the length of BP?

_____ **3.** 4 points in the plane form the vertices of a convex quadrilateral. There are exactly four unique triangles that can be formed using three of four points. If the sum of the areas of these four triangles is 24, what is the area of the quadrilateral?

_____ **4.** A rectangular prism has dimensions $2 \times 3 \times 4$. An octahedron is formed whose vertices are the centers of the faces of the rectangular prism. What is the volume of the octahedron?

_____ **5.** A circle of radius 17 is centered at the origin. Another circle is centered at the point $(21, 0)$, and the two circles share a common chord of length 16. What is the radius of the second circle?

_____ **6.** $YONG$ is a square with side length 10. Let A be the midpoint of YO and B be the midpoint of NG. Let C be the intersection of YB and GA, and D be the intersection of NA and OB. What is the radius of the circle inscribed in rhombus $ABCD$?

_____ **7.** Triangle ABC has $BC = 30$. Point X lies on segment BC such that AX bisects $\angle BAC$. AX is extended past point X and intersects the circumcircle of ABC at another point P different from A. Point Y lies on BC such that PY is perpendicular to BC. If PY has length 8, what is the perimeter of $\triangle BPC$?

_____ **8.** $\triangle ABC$ is a triangle with $AB = 13$, $BC = 14$, $CA = 15$. Let I be the incenter of ABC, G_A be the centroid of $\triangle IBC$, G_B be the centroid of $\triangle ICA$, and G_C be the centroid of $\triangle IAB$. What is the area of triangle $\triangle G_A G_B G_C$? (The incenter is the intersection point of the three angle bisectors and the centroid is the intersection point of the three medians).