1. Frank the Frog is jumping on a row of lilypads numbered 1 through 13 in that order. He can only jump forward 2 or 5 lilypads at a time. Frank is currently on the first lilypad and wishes to reach his home on the thirteenth lilypad. How many ways are there for him to get to his home?

Proposed by Julian Kovalovsky.

Answer: $\boxed{4}$

Solution: Jumping forward 2 lilypads doesn't change whether Frank is on an even or odd numbered lilypad. Jumping forward 5 lilypads will. Frank is trying to go from an odd numbered lilypad to another odd numbered lilypad, so he will need to jump 5 lilypads an even number of times.

- If he never jumps 5 lilypads, there is only one option (jump 2 lilypads 6 times).
- If he jumps 5 lilypads twice, he will also need to jump 2 lilypads once. With three jumps, there are three places in the ordering for him to jump 2 lilypads.

The total is then $1 + 3 = 4$ different ways.

2. Evan wants to become a snapping turtle. A magic genie will turn him into a snapping turtle if he flips two heads in a row with a fair coin. If Evan flips the coin three times, what is the probability that he becomes a snapping turtle?

Proposed by William Zhang.

Answer: $\boxed{\frac{3}{8}}$

Solution: All possible sequences of heads and tails are **HHH**, **HHT**, HTH, HTT, **THH**, THT, TTH, TTT. The bolded items are the cases in which Evan will become a snapping turtle. Counting occurences, the probability is $\frac{3}{8}$.

3. Sam is playing a round of rock paper scissors against a robotic arm. The arm picks randomly between rock, paper, and scissors while Sam picks rock 20% of the time, paper 30% of the time, and scissors 50% of the time. What is the probability Sam wins?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{1}{3}}$

Solution: No matter what Sam chooses, the robot has one choice to win, one choice to tie, and one choice to lose. Each is chosen with $\frac{1}{3}$ chance, so Sam has a $\frac{1}{3}$ chance to win.

4. Michelle flips a coin 9 times in a row and notices 6 flips come up heads. In how many ways can there be three distinct strings of heads of length 1, 2, and 3 in some order? (For example HTHHTHHHT would count)

Proposed by Lewis Lau.

Answer: $\boxed{24}$

Solution: Consider the positions of the tails in the ordering. There must be a tails between adjacent strings of heads, requiring two tails. With one tails left, there are 4 choices on where to order it (before, after, or one of the two locations in between). With $3! = 6$ ways to order the strings of heads, there is a total of $6 \cdot 4 = 24$ possibilities.

5. There are 7 boxes numbered 1 through 7, with 7 balls in each box so that box number x contains x red balls. The rest of the balls in each box are green. A box is then chosen at random and a ball is randomly drawn from it. If the ball is red, what is the probability it came from the box numbered 7?

Proposed by Evan Zhang.

Answer: $\boxed{\dfrac{1}{4}}$

Solution: With the same number of balls in each box, the ball is essentially chosen at random from all 49 balls. Summing, there are $1 + 2 + \cdots + 7 = 28$ red balls. With 7 red balls in box 7, the desired probability is $\frac{7}{28} = \frac{1}{4}$.

6. How many positive integers less than or equal to 300 are divisible by exactly two of 2, 3, and 5? (For example, 12 works because it is divisible by 2 and 3, but not by 5)

Proposed by Lewis Lau.

Answer: $\boxed{70}$

Solution: 2, 3, and 5 are relatively prime, so the desired values are one of:

- divisible by 6, but not 5
- divisible by 10, but not 3
- divisible by 15, but not 2

The total is then calculated as

$$\frac{300}{6}\left(1-\frac{1}{5}\right)+\frac{300}{10}\left(1-\frac{1}{3}\right)+\frac{300}{15}\left(1-\frac{1}{2}\right)=70.$$

7. After combining like terms, how many terms are there in the expansion of $(x+2y+3z+4)^{12}$?

Proposed by Evan Zhang.

Answer: $\boxed{455}$

Solution: All terms are of the form $kx^a(2y)^b(3z)^c4^d$ where k is a nonzero integer and a, b, c, and d are non negative integers with $a+b+c+d=12$. The number of terms is then equal the number of non-negative integer solutions to $a+b+c+d=12$. Using stars and bars gives an answer of $\binom{15}{3}=455$.

8. Mr. Rose has stuffed all subsets of the set $\{1,2,3,4\}$ into a magical hat. Daniel, Leo, and Hannah each pick a subset out of the hat with replacement. What is the probability that Daniel's set is a proper subset of Leo's set, and Leo's set is a proper subset of Hannah's set? (A proper subset of a set S is any subset of S including the empty set but excluding S itself)

Proposed by Daniel He.

Answer: $\boxed{\dfrac{55}{2048}}$

Solution: With 2^4 subsets, there are 2^{12} ways for the three to choose their subsets.

Using the Principle of Inclusion-Exclusion, the number of cases ignoring the *proper* part of the condition is computed first. If the subsets are ordered Daniel's, Leo's, then Hannah's, there will first be some (potentially 0) number of people who don't have 1 in their subset. The remaining in the list will have 1 in their subset. For example, maybe the first two (Daniel and Leo) don't have 1 in their subset. The remaining subsets (Hannah's) will have 1 in them. With 0 to 3 people who have 1 in their subset, there are 4 choices in regards to 1 being in subsets. Across the 4 different numbers, this is 4^4 possibilities.

Now, the cases with at least one pair of equal subsets are subtracted. WLOG, Daniel and Leo's sets are the same. Then, the above idea is repeated for 3^4 possibilities. Additionally accounting for Leo and Hannah having the same set, the total to be subtracted is $2 \cdot 3^4$.

Lastly, the number of cases where all three have the same set must be added back in. This is just the number of subsets, or 2^4.

The final probability is then $\frac{4^4 - 2 \cdot 3^4 + 2^4}{2^{12}} = \frac{55}{2048}$.