MBMT Counting and Probability Round — Weierstrass

March 9, 2025

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1.** Frank the Frog is jumping on a row of lilypads numbered 1 through 13 in that order. He can only jump forward 2 or 5 lilypads at a time. Frank is currently on the first lilypad and wishes to reach his home on the thirteenth lilypad. How many ways are there for him to get to his home?

_____ **2.** Evan wants to become a snapping turtle. A magic genie will turn him into a snapping turtle if he flips two heads in a row with a fair coin. If Evan flips the coin three times, what is the probability that he becomes a snapping turtle?

_____ **3.** Sam is playing a round of rock paper scissors against a robotic arm. The arm picks randomly between rock, paper, and scissors while Sam picks rock 20% of the time, paper 30% of the time, and scissors 50% of the time. What is the probability Sam wins?

_____ **4.** Michelle flips a coin 9 times in a row and notices 6 flips come up heads. In how many ways can there be three distinct strings of heads of length 1, 2, and 3 in some order? (For example HTHHTHHHT would count)

_____ **5.** There are 7 boxes numbered 1 through 7, with 7 balls in each box so that box number x contains x red balls. The rest of the balls in each box are green. A box is then chosen at random and a ball is randomly drawn from it. If the ball is red, what is the probability it came from the box numbered 7?

_____ **6.** How many positive integers less than or equal to 300 are divisible by exactly two of 2, 3, and 5? (For example, 12 works because it is divisible by 2 and 3, but not by 5)

_____ **7.** After combining like terms, how many terms are there in the expansion of $(x + 2y + 3z + 4)^{12}$?

_____ **8.** Mr. Rose has stuffed all subsets of the set $\{1, 2, 3, 4\}$ into a magical hat. Daniel, Leo, and Hannah each pick a subset out of the hat with replacement. What is the probability that Daniel's set is a proper subset of Leo's set, and Leo's set is a proper subset of Hannah's set? (A proper subset of a set S is any subset of S including the empty set but excluding S itself)