1. If $\frac{27}{45} = \sqrt{\frac{9}{x}}$, what is the value of x?

Proposed by William Roe.

Answer: $\boxed{25}$

Solution: Squaring both sides gives us $\frac{9}{x} = \left(\frac{27}{45}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$. Multiplying both sides of the equation by $25x$ gives $9x = 9 \cdot 25$, so $x = 25$.

2. Mr. Bramble teaches a World History class with 30 students. On a quiz, every student initially scored 10 points. Half of the class then retook the quiz, and each student who retook the quiz improved their score by 3 points. By how many points did the class average increase after the retakes?

Proposed by Rakshay Narayanan.

Answer: $\boxed{\dfrac{3}{2}}$

Solution: Before the retakes the class average was 10. After the retakes, 15 students have a score of 10 and 15 students have a score of 13, so the total score among all students is $10 \cdot 15 + 13 \cdot 15 = 345$ and the class average is $\frac{345}{30} = \frac{23}{2}$. So, the class average increased by $\frac{23}{2} - 10 = \frac{3}{2}$ points.

3. Farmer Yunyi has a farm with chickens and chimeras. Chickens have 1 head and 2 legs, and chimeras have 3 heads and 4 legs. If the farm has a total of 200 heads and 350 legs (not counting Farmer Yunyi), how many animals are on the farm?

Proposed by Evan Zhang.

Answer: $\boxed{150}$

Solution: Let c represent the number of chickens on Farmer Yunyi's farm, and let m represent the number of chimeras on Farmer Yunyi's farm. We have

$$c + 3m = 200$$

$$2c + 4m = 350$$

Subtracting the two equations gives $c + m = 150$, so there are 150 animals on the farm.

4. Pedro is trying to find rectangles with positive integer side lengths whose areas are numerically equal to their perimeters. To his surprise, he discovers that there are only two unique rectangles that have this property. What is the sum of the areas of these two rectangles?

Proposed by Evan Zhang.

Answer: $\boxed{34}$

Solution: Let the side lengths of the rectangle be a and b. Our rectangles satisfy the equation $ab = 2a + 2b$, which we can factor into $(a - 2)(b - 2) = 4$.
Since a and b are both positive, then either $(a - 2)$ and $(b - 2)$ are 1 and 4 in some order, or are 2 and 2 in some order. are two rectangles are a 3×6 rectangle and a 4×4 rectangle. The sum of the areas of these two rectangles is $3 \cdot 6 + 4 \cdot 4 = 18 + 16 = 34$.

5. A falling ping-pong ball always bounces up to $\frac{3}{4}$ of the height it falls from. If Stanley drops a ball from 40 feet above the ground, what is the total distance the ball will travel?

Proposed by Evan Zhang.

Answer: $\boxed{280}$

Solution: The ball travels 40 feet during the first fall, then a total of $2\left(\frac{3}{4} \cdot 40\right)$ feet during the first bounce and resulting fall, then a total of $2\left(\frac{3}{4}\right)^2 \cdot 40$ feet during the second bounce and resulting fall, and so on.

The total distance the ball travels is then

$$40 + 2\left(\frac{3}{4} \cdot 40 + \left(\frac{3}{4}\right)^2 \cdot 40 + \cdots\right) = 40 + 2\left(\frac{30}{1 - \frac{3}{4}}\right) = 40 + 2 \cdot 120 = 280.$$

6. If $\frac{1}{x^3} + \frac{1}{y^3} = 4$ and $\frac{1}{x} + \frac{1}{y} = 2$, what is $\frac{1}{x^2} + \frac{1}{y^2}$?

Proposed by Valerie Song.

Answer: $\boxed{\dfrac{8}{3}}$

Solution: Cubing $\frac{1}{x}+\frac{1}{y}=2$ gives $\frac{1}{x^3}+\frac{3}{x^2y}+\frac{3}{xy^2}+\frac{1}{y^3}=8$. Since $\frac{1}{x^3}+\frac{1}{y^3}=4$, we know $4+\frac{3}{xy}\left(\frac{1}{x}+\frac{1}{y}\right)=8$, so $\frac{6}{xy}=4$ and $\frac{1}{xy}=\frac{2}{3}$.

Squaring $\left(\frac{1}{x}+\frac{1}{y}\right)=2$ gives $\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=4$, so $\frac{1}{x^2}+\frac{1}{y^2}=4-\frac{2}{xy}=4-\frac{4}{3}=\frac{8}{3}$.

7. Michael takes the polynomial $x^{101}+1$ and divides it by $x+1$, giving him the polynomial $f(x)$. He then takes $f(x)$ and divides it by $x+1$ again, giving him the polynomial $g(x)$ and the remainder s. What is the value s?

Proposed by Daniel He.

Answer: $\boxed{101}$

Solution: To find $\frac{x^{101}+1}{x+1}$, we have $\frac{x^{101}}{x}=x^{100}$, so $\frac{x^{101}+1}{x+1}=x^{100}+\frac{-x^{100}+1}{x+1}$. Then, $\frac{x^{100}}{x}=x^{99}$, so $\frac{x^{101}+1}{x+1}=x^{100}-x^{99}+\frac{x^{99}+1}{x+1}$. We can continue this pattern to get

$$\frac{x^{101}+1}{x+1}=x^{100}-x^{99}+x^{98}-x^{97}\cdots-x+1.$$

Let $f(x)=g(x)(x-a)+r$ such that r is the remainder when dividing $f(x)$ by $x-a$ and $g(x)$ is the quotient. Plugging in $x=a$ shows $r=f(a)$. With this, we find $f(-1)=1+1+1+\cdots+1=101$.

8. A sequence of functions is defined as follows: $f_0(x)=||x|-1|$ and $f_n(x)=|f_{n-1}(x)-2^n|$ for all positive integers n. Let x_n be the positive root of $f_n(x)$; in other words, $x_n>0$ and $f_n(x_n)=0$. What is the sum of x_n as n goes from 0 to 2025, inclusive?

Proposed by Evan Zhang.

Answer: $\boxed{2^{2027}-2028}$

Solution: To find x_0, we need to solve for $||x_0|-1|=0$, so $|x_0|=1$, and $x_0=1$.

To find x_1, we need to solve for $|||x_1|-1|-2|=0$, so $||x_1|-1|=2$. Since $|x_1|$ is positive, $|x_1|-1=2$, so $x_1=1+2$.

If we continue this, we see that in general, $x_n=1+2+4+\cdots+2^n=2^{n+1}-1$. So, the sum of x_n as n goes from 0 to 2025 is $(2^1-1)+(2^2-1)+\cdots+(2^{2026}-1)=(2^1+2^2+\cdots+2^{2026})-2026=2^{2027}-2028$.