1. What is the smallest positive three-digit integer that is a multiple of 5, but not a multiple of 2 or 3?

Proposed by Lewis Lau.

Answer: $\boxed{115}$

Solution: The smallest three-digit multiple of 5 is 100, but that is divisible by 2. Next up is 105, but that is divisible by 3. 110 is likewise divisible by 2, but 115 is not divisible by 2 or 3, so that is our answer.

2. What is the second-smallest positive integer that is a multiple of both 4 and 6?

Proposed by Lewis Lau.

Answer: $\boxed{24}$

Solution: The smallest positive integer that is divisible by both 4 and 6 is the LCM, which is 12. The next smallest is $2 \cdot 12 = 24$.

3. Evan bakes seventy cookies. He can put the cookies in bags with either six cookies or ten cookies per bag. How many more *completely full* bags would he have if he put six cookies in each bag than if he put ten cookies in each bag?

Proposed by Jason Youm.

Answer: $\boxed{4}$

Solution: If Evan put cookies in bags of 6, he would have 11 full bags with 4 cookies left over. If he put cookies in bags of 10, he would have 7 full bags with no cookies left over. The difference is therefore $11 - 7 = 4$.

4. What is the smallest positive integer N such that its value is 5 times the sum of its digits?

Proposed by Ivy Guo.

Answer: $\boxed{45}$

Solution: Clearly N can not have only a single digit, because otherwise $5N = N$, but N is positive.

If N is a two-digit number, let $N = 10a + b$, so $10a + b = 5(a + b)$. This rearranges to $5a = 4b$. a and b are both digits, so $a = 4$ and $b = 5$.

5. Alice's answer to her math homework has been eaten by her pet ants, who only eat their favorite digit. Her answer is now $7X91X8$ where X is a missing digit. If Alice remembers that her answer was divisible by 12, what digit did the ants eat?

Proposed by Ashley Zhang.

Answer: $\boxed{4}$

Solution: Alice's answer must be divisible by 3. Summing up all the digits of her answer gives $25 + 2X$, which must divide 3. The only digits X for which this works are 1, 4, and 7. Alice's answer must additionally divide 4, which means $X8$ must divide 4. The only possible value for X that satisfies both conditions is $X = 4$.

6. Shriyan divides his favorite three-digit number by 2, 3, 4, 8, 9, and 11 and gets a remainder of 1 each time. What is Shriyan's favorite three-digit number?

Proposed by Jason Youm.

Answer: $\boxed{793}$

Solution: Let us find the least common multiple of 2, 3, 4, 8, 9, and 11. We need to have 3 factors of 2, 2 factors of 3, and 1 factor of 11. Multiplying these out gives a value of 792, so if Shriyan's number is 793, it will have a remainder of 1 when divided by each of 2, 3, 4, 8, 9, and 11.

7. Three *consecutive* nonzero digits are taken, and the 6 numbers formed by permuting the digits are added. What is the largest integer that must divide the sum?

Proposed by Evan Zhang.

Answer: $\boxed{666}$

Solution: Let the digits be $n-1$, n, and $n+1$ where $2 \leq n \leq 8$. Each digit appears in each location twice, so the sum is $200((n-1)+n+(n+1))+20((n-1)+n+(n+1))+2((n-1)+n+(n+1))$. This becomes $222 \cdot 3n = 666n$. 666 clearly divides this. Trying both $n = 2$ and $n = 3$ shows 666 is the largest number that will always divide $666n$.

8. Let $\lfloor x \rfloor$ represent the largest integer less than or equal to x. There exists a unique 5-digit positive integer n such that the sum of its digits is 20 and

$$\left\lfloor \frac{n}{10} \right\rfloor + \left\lfloor \frac{n}{100} \right\rfloor + \left\lfloor \frac{n}{1000} \right\rfloor + \left\lfloor \frac{n}{10000} \right\rfloor = 2025$$

What is the product of the digits of n?

Proposed by Chaewoon Kyoung.

Answer: $\boxed{320}$

Solution: Let $n = \underline{a_1a_2a_3a_4a_5} = a_1 \cdot 10^4 + a_2 \cdot 10^3 + a_3 \cdot 10^2 + a_4 \cdot 10^1 + a_5$, where $a_i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 0\}$ for $i = 1, 2, 3, 4, 5$, and $a_1 \neq 0$. Then,

$$\begin{aligned}
\left\lfloor \frac{n}{10} \right\rfloor &= a_1 \cdot 10^3 + a_2 \cdot 10^2 + a_3 \cdot 10^1 + a_4, \\
\left\lfloor \frac{n}{100} \right\rfloor &= a_1 \cdot 10^2 + a_2 \cdot 10^1 + a_3, \\
\left\lfloor \frac{n}{1000} \right\rfloor &= a_1 \cdot 10^1 + a_2, \text{and} \\
\left\lfloor \frac{n}{10000} \right\rfloor &= a_1.
\end{aligned}$$

The equation then becomes $a_1 \cdot 1111 + a_2 \cdot 111 + a_3 \cdot 11 + a_4 = 2025$. Since $2 \cdot 1111 > 2025, a_1 < 2$ and therefore $a_1 = 1$.

Then, $a_2 \cdot 111 + a_3 \cdot 11 + a_4 = 2025 - 1111 = 914$. The maximum of $a_3 \cdot 11 + a_4$ is $9 \cdot 11 + 9 = 108$, so $a_2 \cdot 111 \geq 914 - 108 = 806$. Therefore, $a_2 >= 8$.

Also, since $9 \cdot 111 > 914$, $a_2 < 9$ and thus $a_2 = 8$. Then, $a_3 \cdot 11 + a_4 = 914 - 888 = 26$. Since the maximum of a_4 is 9, $a_3 \cdot 11 \geq 26 - 9 = 17$, so $a_3 \geq 2$. Also, since $3 \cdot 11 > 26$, $a_3 < 3$ and thus $a_3 = 2$.

Then $a_4 = 26 - 22 = 4$.

Since $a_1 + a_2 + a_3 + a_4 + a_5 = 20$ is given in the problem, $a_5 = 20 - a_1 - a_2 - a_3 - a_4 = 20 - 1 - 8 - 2 - 4 = 5$. Finally, the product of all digits are $a_1a_2a_3a_4a_5 = 1 \cdot 8 \cdot 2 \cdot 4 \cdot 5 = 320$.