MBMT Erdös Guts Round — Set 1

March 9, 2025

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1. **[3]** A square and an equilateral triangle both have perimeter 48. What is the difference between the side length of the triangle and the side length of the square?

2. **[3]** What is the smallest perfect square which ends in the digit 5?

3. **[3]** What is $2025^{20^{2^{2^5}}}$?

4. **[3]** The probability of getting heads when flipping an unfair coin is twice the probability of getting tails when flipping the same coin. What is the probability of getting heads on one flip of the unfair coin?

5. **[3]** Bob eats 5 bananas for every 2 bananas Kevin eats. If Bob and Kevin eat 42 bananas together, how many more bananas did Bob eat than Kevin?

MBMT Erdös Guts Round — Set 2

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6. **[4]** Which non-zero digit x satisfies the property that the three-digit number $\overline{x01}$ is divisible by 7?

7. **[4]** Two trucks, one of length 7 feet and the other of length 11 feet, are passing each other in opposite directions. If the first truck is driving at 6 feet per second and 2 seconds elapse from when the trucks begin passing each other to when they are fully past each other, how fast is the second truck traveling, in feet per second?

8. **[4]** What is the maximum number of intersection points between two squares?

9. **[4]** x and y are real numbers. Gloria the Cicada wants to estimate the value of $x - y$, so she rounds x down by 0.05 and y up by 0.02. What is the positive difference between Gloria the Cicada's estimated value and her actual value?

10. **[4]** Jason draws a rocket. Sadly, he is very bad at drawing, so his rocket simply consists of an isosceles triangle sharing its base with one side of a rectangle. Given that each side of the rocket is an integer, if the rectangle has perimeter 40 and the isosceles triangle has perimeter 24, what is the largest possible perimeter of Jason's rocket?

MBMT Erdös Guts Round — Set 3

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11. **[5]** Kian is buying pizza. A pizza with a diameter of 5 inches costs him 100 dollars. How much should a pizza with the same thickness and a diameter of 12 inches cost to have the same amount of pizza per dollar?

12. **[5]** What is the sum of the cubes of the first four prime numbers?

13. **[5]** Evan was biking to RTC, but $\frac{2}{3}$ of the way there, his bike broke down, so he walked the rest of the way there. If he walked for $\frac{2}{3}$ of the total time, how many times faster is his biking speed than his walking speed?

14. **[5]** A hexagon is inscribed in a circle. Yunyi draws two triangles, using each vertex of the hexagon exactly once. What is the probability that the two triangles intersect?

15. **[5]** Jimmy's least favorite number is between 0 and 64. He decides to add up all the numbers from 0 to 64, leaving out his least favorite number. If he correctly finds the sum to be 2025, what is Jimmy's least favorite number?

MBMT Erdös Guts Round — Set 4

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16. [7] If $x^2 = (7 \cdot 9 + 1)(13 \cdot 15 + 1)(19 \cdot 21 + 1)$, what is the value of x?

17. [7] Cola has a playlist they love to listen to in their free time. They listen to their playlist starting from the first song, and then continuously loop through the playlist. At the 70th song of their music listening session, Cola is listening to the first song again. If there were a prime number of songs in the playlist and each song was 4 minutes long, what was the maximum length of one loop of the playlist?

18. [7] Yunyi has a cube and notices that its volume and surface area have the same numerical value a. Kian has a sphere and also notices that its volume and surface area have the same numerical value b. What is $\frac{a}{b}$?

19. [7] A frog is at the point $(0,0)$ on the coordinate plane and spots a fly at the point $(4,2)$. At each step, if the frog is at the point (x,y), it can move to the points $(x+1,y)$, $(x,y+1)$, or $(x+1,y+1)$. How many sequences of moves are there for the frog to reach the fly?

20. [7] Leroy the Leg Lover lords over his legendary legion of leggy pets. Soggy squids have 10 legs each and outstanding octopi have 8 legs each. If Leroy has more squids than octopi, and his leggy total tallies to 134, how many animals does he have?

MBMT Erdös Guts Round — Set 5

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21. **[9]** A circle with unknown radius passes through vertices A and B of rectangle $ABCD$. The circle is tangent to side DC and intersects side BC at E so that $BE = 10$ and $CE = 8$. What is the length of side AB?

22. **[9]** What is the 100th smallest positive integer that contains only odd digits?

23. **[9]** Triangle ABC has $AB = 6$, $BC = 8$, and $AC = 10$. If Alice chooses a random point within the triangle, what is the probability that the point lies in the incircle of ABC?

24. **[9]** For a positive integer n, let $f(n)$ represent the largest prime number that divides $n!$. What is the sum of $f(n)$ as n goes from 2 to 13, inclusive?

25. **[9]** Stephen rolls a fair six sided die. He then randomly paints a number of faces of a cube corresponding to the number shown on the die (if he rolls a 4, he paints 4 random faces). Stephen then cuts the cube into 64 smaller cubes and picks one at random to roll. What is the probability that the top face of the small cube is painted?

MBMT Erdös Guts Round — Set 6

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26. [12] Let $f(x)$ be the product

$$\left(1 + \frac{1}{10^6}\right)\left(1 + \frac{2}{10^6}\right)\cdots\left(1 + \frac{x}{10^6}\right).$$

Estimate the smallest positive integer x such that $f(x) > 10^6$.

27. [12] Square $A_0 B_0 C_0 D_0$ has a side length of 1. For all positive integers n, square $A_n B_n C_n D_n$ is formed by inscribing an equilateral triangle in square $A_{n-1} B_{n-1} C_{n-1} D_{n-1}$ such that a vertex of the triangle coincides with a vertex of the square, and then inscribing a square in the equilateral triangle. Find

$$\sum_{i=0}^{\infty}[A_i B_i C_i D_i]$$

where $[A_i B_i C_i D_i]$ denotes the area of square $A_i B_i C_i D_i$.

28. [12] 3 random integers a, b, and c are chosen with replacement from -2025 to 2025 inclusive. Estimate the probability that the quadratic $ax^2 + bx + c$ has at least 1 real root.

29. [12] Yunyi has a circular table with a radius of 2025 inches. He also has an infinite number of stickers in the shape of a circle with a radius of 1 inch. If stickers can overlap and hang off the edge of the table, what is the minimum number of stickers Yunyi needs to cover the entirety of the circle's surface with stickers?

30. [12] Kian is in a room with 2025 lightbulbs which are all initially off. Every second he picks one of the lightbulbs at random and flips its state (if it was off initially, he turns it on, and vice versa). Estimate the floor of the log of the expected number of seconds it'll take for him to turn all of the lightbulbs on.