1. What is the ratio between the area and circumference of a circle with a radius of 10?

Proposed by Evan Zhang.

Answer: $\boxed{5}$

Solution: The area of a circle with radius r is πr^2, and the circumference is $2\pi r$. The ratio would therefore be $\frac{\pi r^2}{2\pi r} = \frac{r}{2}$. Plugging in $r = 10$ gives an answer of 5.

2. Three points A, T, and W lie on a circle. A and W are the endpoints of the circle's diameter, and T is equidistant from A and W. What is the measure, in degrees, of $\angle ATW$?

Proposed by Lewis Lau.

Answer: $\boxed{90^\circ}$

Solution: The measure of an inscribed angle is half the angle measure of the subtended arc. Since AW is a diameter, the subtended arc measures 180°, so $\angle ATW = 90^\circ$.

3. A triangle has two sides of length 5 and 6. What is the maximum possible area for such a triangle?

Proposed by Evan Zhang.

Answer: $\boxed{15}$

Solution: If we drop the height to the side with length 5, we can see it is one of the legs of a right triangle with the side of length 6 as a hypotenuse. The legs of a (possibly degenerate) right triangle are always less than or equal to the hypotenuse, so to maximize the height (and therefore the area) we want the side of length 6 to be perpendicular to the side of length 5, maximizing the height. This gives an area of $\frac{5 \cdot 6}{2} = 15$.

4. 4 points in the plane form the vertices of a convex quadrilateral. There are exactly four unique triangles that can be formed using three of four points. If the sum of the areas of these four triangles is 24, what is the area of the quadrilateral?

Proposed by Sam Easaw.

Answer: $\boxed{12}$

Solution: If we draw out the two diagonals, we split the quadrilateral into 4 smaller triangles. Each of the unique triangles in the problem are made up of 2 of these smaller triangles. Adding up the areas of the 4 unique triangles adds each small triangle twice, as each small triangle is contained within two of the unique triangles. The area of the quadrilateral is the area of the small triangles, which is $\frac{24}{2} = 12$.

5. A rectangular prism has dimensions $2 \times 3 \times 4$. An octahedron is formed whose vertices are the centers of the faces of the rectangular prism. What is the volume of the octahedron?

Proposed by Ivy Guo.

Answer: $\boxed{4}$

Solution: Consider cutting the rectangular prism into 8 octants; the piece of the octahedron in each octant is a tetrahedron that shares 3 sides with the sides of the octant.

The volume of a pyramid is $\frac{1}{3}bh$, where b is the area of the base and h is the height. The base is a right triangle with legs 2 and 1, so its area is $\frac{1}{2} \cdot 2 \cdot 1 = 1$. The height is 1.5. Therefore, the volume of the tetrahedron is $\frac{1}{3} \cdot 1.5 = \frac{1}{2}$.

There are 8 of these tetrahedrons, one in each octant, so the total volume is $8 \cdot 0.5 = 4$.

6. Rectangle $ABCD$ has side lengths $AB = 2$ and $BC = 1$. E and F are the midpoints of AB and CD, respectively. A circle is drawn with center F that passes through points E and C. Another circle is drawn with center D and passes through points A and F. What is the area of the region bounded by AE, FD, arc AF, and arc ED?

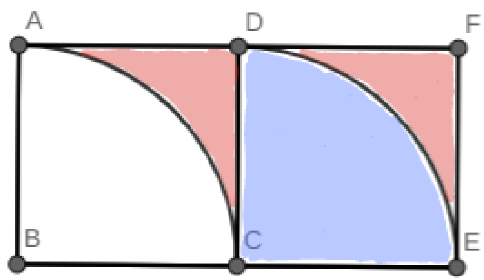
The original problem statement was incorrect. The revised version is here:

Rectangle $ABCD$ has side lengths $AB = 2$ and $BC = 1$. E and F are the midpoints of AB and CD, respectively. A circle is drawn with center F that passes through points E and C. Another circle is drawn with center D and passes through points A and F. What is the area of the region bounded by AE, FC, arc AF, and arc EC?

Proposed by Olivia Guo.

Answer: $\boxed{1}$

Solution:



We can split the region into two parts, one bounded by AE, EF, and arc AF, and one bounded by EF, FC, and arc EC. Notice that the first region is identical to the region bounded by EB, BC, and arc EC, so the area of the region of interest is the same as the area of square $EBCF$. Since each side of this square has side length 1, the area of the region of interest is 1.

7. A circle of radius 17 is centered at the origin. Another circle is centered at the point $(21, 0)$, and the two circles share a common chord of length 16. What is the radius of the second circle?

Proposed by Evan Zhang.

Answer: $\boxed{10}$

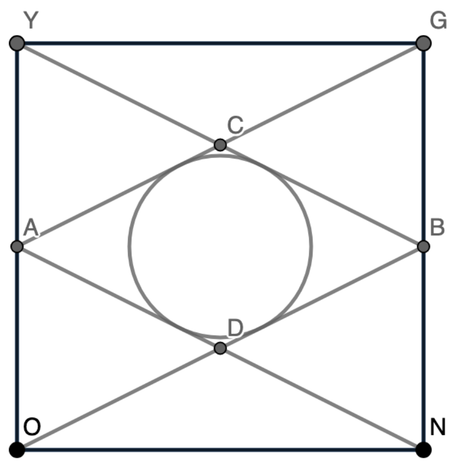
Solution: Let the centers of the two circles be O_1 and O_2, and let the common chord have endpoints A and B. Let P be the point of intersection between AB and O_1O_2. Since $\triangle O_1PA$ is right, we know $O_1P = 15$ by the Pythagorean Theorem. This gives $O_2P = 21 - 15 = 6$. Since O_2PA is also right, by the Pythagorean Theorem again, $O_2A = \sqrt{6^2 + 8^2} = 10$.

8. $YONG$ is a square with side length 10. Let A be the midpoint of YO and B be the midpoint of NG. Let C be the intersection of YB and GA, and D be the intersection of NA and OB. What is the radius of the circle inscribed in rhombus $ABCD$?

Proposed by Evan Zhang.

Answer: $\boxed{\sqrt{5}}$

Solution:



We can solve this by finding the area of the rhombus in two ways. First of all, since $\triangle BCD$ is similar to $\triangle BYO$ in a $1:2$ ratio, $CD = 5$. Since $AB = 10$, the area is $\frac{5\cdot 10}{2} = 25$.

By the Pythagorean Theorem, $AG = \sqrt{5^2 + 10^2} = 5\sqrt{5}$. Thus, $AC = \frac{5\sqrt{5}}{2}$. If the inradius is r, we can split the rhombus into four triangles to get a total area of $2r \cdot AC = 5\sqrt{5} \cdot r$.

Setting the two areas equal gives $5\sqrt{5} \cdot r = 25$, so $r = \sqrt{5}$.