1. Veena is selling three pots colored purple, turquoise, and orange. If she sold the purple pot sometime after the orange pot, in how many possible orders could she have sold the pots?

Proposed by Reanna Jin.

Answer: $\boxed{3}$

Solution: There are $3! = 6$ permutations of these three pots. Listing out the permutations and counting them, we find that three permutations have the purple pot after the orange pot.

Symmetry can also be used. Since the purple pot is either before or after the orange pot and both outcomes have equal probability, we get $\frac{1}{2} \cdot 6 = 3$.

2. Frank the Frog is jumping on a row of lilypads numbered 1 through 13 in that order. He can only jump forward 2 or 5 lilypads at a time. Frank is currently on the first lilypad and wishes to reach his home on the thirteenth lilypad. How many ways are there for him to get to his home?

Proposed by Julian Kovalovsky.

Answer: $\boxed{4}$

Solution: Jumping forward 2 lilypads doesn't change whether Frank is on an even or odd numbered lilypad. Jumping forward 5 lilypads will. Frank is trying to go from an odd numbered lilypad to another odd numbered lilypad, so he will need to jump 5 lilypads an even number of times.

- If he never jumps 5 lilypads, there is only one option (jump 2 lilypads 6 times).
- If he jumps 5 lilypads twice, he will also need to jump 2 lilypads once. With three jumps, there are three places in the ordering for him to jump 2 lilypads.

The total is then $1 + 3 = 4$ different ways.

3. Evan wants to become a snapping turtle. A magic genie will turn him into a snapping turtle if he flips two heads in a row with a fair coin. If Evan flips the coin three times, what is the probability that he becomes a snapping turtle?

Proposed by William Zhang.

Answer: $\boxed{\frac{3}{8}}$

Solution: All possible sequences of heads and tails are **HHH**, **HHT**, HTH, HTT, **THH**, THT, TTH, TTT. The bolded items are the cases in which Evan will become a snapping turtle. Counting occurences, the probability is $\frac{3}{8}$.

4. Sam is playing a round of rock paper scissors against a robotic arm. The arm picks randomly between rock, paper, and scissors while Sam picks rock 20% of the time, paper 30% of the time, and scissors 50% of the time. What is the probability Sam wins?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{1}{3}}$

Solution: No matter what Sam chooses, the robot has one choice to win, one choice to tie, and one choice to lose. Each is chosen with $\frac{1}{3}$ chance, so Sam has a $\frac{1}{3}$ chance to win.

5. Pete the Cat enjoys dressing tastefully to impress. He has 4 capes, 3 jackets, and 5 hats, all of which are distinct, and he can only wear up to one of each type of clothing. A tasteful outfit consists of at least 2 different pieces of clothing. How many distinct tasteful outfits can Pete the Cat make?

Proposed by Ashley Zhang.

Answer: $\boxed{107}$

Solution: If Pete the Cat only wears 2 pieces of clothing, he has $3\times4+3\times5+4\times5 = 47$ choices (each term corresponds to one choice of clothing item to not wear). If Pete wears 3 pieces of clothing, he has $3 \times 4 \times 5 = 60$ choices. Adding gives $60 + 47 = 107$ total tasteful outfits.

6. Michelle flips a coin 9 times in a row and notices 6 flips come up heads. In how many ways can there be three distinct strings of heads of length 1, 2, and 3 in some order? (For example HTHHTHHHT would count)

Proposed by Lewis Lau.

Answer: $\boxed{24}$

Solution: Consider the positions of the tails in the ordering. There must be a tails between adjacent strings of heads, requiring two tails. With one tails left, there are 4 choices on where to order it (before, after, or one of the two locations in between). With $3! = 6$ ways to order the strings of heads, there is a total of $6 \cdot 4 = 24$ possibilities.

7. There are 7 boxes numbered 1 through 7, with 7 balls in each box so that box number x contains x red balls. The rest of the balls in each box are green. A box is then chosen at random and a ball is randomly drawn from it. If the ball is red, what is the probability it came from the box numbered 7?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{1}{4}}$

Solution: With the same number of balls in each box, the ball is essentially chosen at random from all 49 balls. Summing, there are $1 + 2 + \cdots + 7 = 28$ red balls. With 7 red balls in box 7, the desired probability is $\frac{7}{28} = \frac{1}{4}$.

8. How many positive integers less than or equal to 300 are divisible by exactly two of 2, 3, and 5? (For example, 12 works because it is divisible by 2 and 3, but not by 5)

Proposed by Lewis Lau.

Answer: $\boxed{70}$

Solution: 2, 3, and 5 are relatively prime, so the desired values are one of:

- divisible by 6, but not 5
- divisible by 10, but not 3
- divisible by 15, but not 2

The total is then calculated as

$$\frac{300}{6}\left(1 - \frac{1}{5}\right) + \frac{300}{10}\left(1 - \frac{1}{3}\right) + \frac{300}{15}\left(1 - \frac{1}{2}\right) = 70.$$