MBMT Team Round — Gödel

April 16, 2023

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

MBMT Team Round Answer Sheet — Gödel

April 16, 2023

Team Name _____

Team Number _____

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1 **[4]** Anne wants to crochet a rectangular blanket with 36 square meters of yarn. If the side lengths of her blanket must be positive integers and she uses up all of her yarn in the process - how many different widths can her blanket have?

2 **[4]** Timmy is playing a game where he is given an integer x amount of money between 1 and 100 inclusive, but then has a $x\%$ chance of losing the money. What amount of money should Timmy choose to maximize his profit?

3 **[4]** How many 2-digit primes have digits summing to a prime?

4 **[5]** How many 4-digit integers have digits multiplying to 24?

5 **[5]** In right triangle $\triangle$ ABC, angle C is 90 degrees and AC = 10. If angle A is between 45 and 60 degrees, exclusive, how many integer values could side length BC possibly be?

6 **[5]** The circle $(x-5)^2+(y-3)^2=16$ is tangent to the line $y=x+b$. Find the sum of all possible values of b.

7 **[6]** If r_1 and r_2 are roots for the quadratic x^2-7x+5, find ${r_1}^3+{r_2}^3$.

8 **[6]** How many lattice points lie on the interior of the figure described by the equation $(|2x|+|3y|-12)^2<36$?

9 **[7]** Find the smallest positive integer k such that $k\cdot(2^2-1)(3^2-1)(4^2-1)...(2024^2-1)$ is a perfect square.

10 **[8]** We define the distance between two vertices on a regular icosahedron to be the least number of edges to be traversed to get from one point to the other. An ant is at vertex A at time 0. Every second, the ant chooses a random vertex adjacent to its current vertex (one edge away) and moves to it. What is the probability the ant will be at a distance of 2 from vertex A after 4 seconds?

11 **[8]** Define an operation $x \ \& \ y = x + y^2$. What is $((...(((1 \ \& \ 3) \ \& \ 5) \ \& \ ...) \ \& \ 19)$?

12 **[9]** Let A be the greatest lower bound of the smallest possible angle in an equilateral, convex pentagon. Find sin(A).

13 **[9]** Given a hexagon, how many ways are there to paint all its corners either red or blue? Rotations and reflections count as identical hexagons.

14 **[10]** Evaluate $9^{9^{9!}} \pmod{5!}$

15 **[10]** In triangle ABC, $AB = 5$, $AC = 6$, and $BC = 7$. There exists a point P for which the distance from P to AB is 2 and the distance from P to AC is 3. Find the sum of all possible distances from P to BC.