1 If the Earth really was flat and a circle with radius 4 miles, and 75% of the surface area was ocean, what is the surface area NOT covered by ocean in square miles?

Answer: $\boxed{4\pi}$

Solution: The surface area is $\pi \cdot (4)^2 = 16\pi$. Since 75% of the surface area is ocean, 25% of it is not ocean, so $16\pi \cdot \frac{1}{4} = 4\pi$ square miles is not covered by ocean.

2 Maggie wants to paint her bedroom blue-gray. To make blue-gray paint, she must mix one part blue paint with two parts white paint. If she has 12 liters of white paint, how much blue-gray paint will she have after she mixes in the right amount of blue paint?

Answer: $\boxed{18}$

Solution: Because Maggie has 12 liters of white paint, she should mix in $\frac{12}{2} = 6$ liters of blue paint. So, Maggie will have $12 + 6 = 18$ liters of blue-gray paint.

3 Let t be the answer to this question. If $x^2 + ax + b$ has exactly one real solution, t, and $a \neq b$, find the value of b.

Answer: $\boxed{1}$

Solution: Since $x^2 + ax + b$ has exactly one real solution, the discriminant, $a^2 - 4b$, must equal 0, and the solution, t, is $\frac{a}{2}$ from the quadratic formula. We also know $b = t$. So, $4t^2 - 4t = 0$, so $t = 1$ or $t = 0$. Since $a \neq b$, $t = 1$.

4 Bob has b roses. Bobeth has double the number of roses that Bob has. Boberta has $\frac{1}{4}b^2$ more roses than Bob. How many roses do all three of them have together if Bobeth and Boberta have the same number of roses? They all have at least one rose.

Answer: $\boxed{20}$

Solution: Since Bobeth and Boberta have the same number of roses, $2b = b + \frac{1}{4}b^2 \longrightarrow b = \frac{1}{4}b^2 \longrightarrow 1 = \frac{1}{4}b \longrightarrow b = 4$. So, the three of them have $4 + 8 + 8 = 20$ roses together.

5 Bonnie likes to collect stickers. On her birthday, she collects the same number of stickers as the age she is turning. While she was not given any before she was 5, she started by collecting 5 stickers at her 5th birthday. How many stickers will she have in total when she turns 24, including those she collects at her 24th birthday?

Answer: $\boxed{290}$

Solution: We want to find $5 + 6 + 7 + ... + 23 + 24$. Using the formula for the sum of an arithmetic sequence, this is equal to $\frac{5+24}{2} \cdot (24 - 5 + 1) = 290$.

6 How many numbers less than or equal to 75 are divisible by 2, 3, or 7?

Answer: $\boxed{53}$

Solution: We'll use principle of inclusion-exclusion.

There are $\lfloor \frac{75}{2} \rfloor = 37$ numbers divisible by 2, $\lfloor \frac{75}{3} \rfloor = 25$ numbers divisible by 3, and $\lfloor \frac{75}{7} \rfloor = 10$ numbers divisible by 7.

Numbers divisible by two of 2, 3, and 7 were counted twice, so we must subtract the $\lfloor \frac{75}{2\cdot 3} \rfloor = 12$ divisible by 2 and 3, the $\lfloor \frac{75}{2\cdot 7} \rfloor = 5$ divisible by 2 and 7, and the $\lfloor \frac{75}{3\cdot 7} \rfloor = 3$ divisible by 3 and 7. Finally, numbers divisible by 2, 3, and 7 aren't being counted, so we must add the the $\lfloor \frac{75}{2\cdot 3\cdot 7} \rfloor = 1$ divisible by 2, 3 and 7.

The total amount of numbers is $37 + 25 + 10 - 12 - 5 - 3 + 1 = 53$.

7 Find the area of the largest equilateral triangle that can be inscribed in a unit square.

Answer: $\boxed{2\sqrt{3} - 3}$

Solution: Let our unit square be $ABCD$, and the equilateral triangle be AEF, such that E is on BC and F is on CD. By symmetry, the largest equilateral triangle is the one such that $\angle EAB = \angle DAF = 15°$.

So, $AE = \frac{1}{\cos 15} = \frac{1}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \sqrt{6} - \sqrt{2}$, so the area of $\triangle AEF$ is $\frac{(\sqrt{6}-\sqrt{2})^2\sqrt{3}}{4} = 2\sqrt{3} - 3$.

8 Taylor drops a surprise album from a height of 13 meters. If the height of each bounce is 2/3 the height of the previous bounce, find the total vertical distance traveled by the album.

Answer: $\boxed{65}$

Solution: The first drop travels a distance of 13 meters, the first bounce and drop travels a total distance of $\frac{2}{3} \cdot 26$, the second bounce and drop travels a total distance of $\left(\frac{2}{3}\right)^2 \cdot 26$, and so on. So, we want to find

$$13 + 26\left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 ...\right).$$

This is equal to

$$13 + 26\left(\frac{\frac{2}{3}}{1 - \frac{2}{3}}\right) = 13 + 26 \cdot 2 = 65 \text{ meters.}$$

9 Bob can do 5 pages of math homework in 3 hours. Jeff can do 5 pages of math homework in 4 hours. If Bob and Jeff worked together, how long would it take them to finish 35 pages of math homework?

Answer: $\boxed{12}$

Solution: In one hour, Bob can do $\frac{5}{3}$ pages of math homework, and Jeff can do $\frac{5}{4}$ pages of math homework. So, together, they can do $\frac{5}{3} + \frac{5}{4} = \frac{35}{12}$ pages of math homework in one hour. So, it would take then $\frac{35}{\frac{35}{12}} = 12$ hours to do 35 pages of math homework.

10 A "crescent" is made by putting a smaller circle inside a larger circle (shown below). The smaller circle's radius is 2/3 of the larger circle's radius. If the larger circle has radius 9, what is the area of the crescent (shaded region)?

Answer: $\boxed{45\pi}$

Solution: The area of the larger circle is $\pi \cdot 9^2 = 81\pi$, and the area of the smaller circle is $\pi \cdot \left(\frac{2}{3} \cdot 9\right)^2 = 36\pi$. So, the area of the crescent is $81\pi - 36\pi = 45\pi$.

11 How many ways are there to tile a 99x102 grid with octominoes the shape of 3x3 blocks with a single missing corner? A grid is considered tiled if it is covered with no gaps.

Answer: $\boxed{0}$

Solution: Each octomino covers 8 squares, and there are a total of 10098 squares in the grid. Since 10098 is not divisible by 8, it is impossible to tile the grid with octominoes.

12 In a parallelogram ABCD, AB = 4, BC = 7, and ∠ ABC = 75°. Find AC^2+ BD^2.

Answer: $\boxed{130}$

Solution: By Law of Cosines, $AC^2 = 4^2 + 7^2 - 4 \cdot 7 \cdot \cos 75 = 16 + 49 - 28 \cdot \frac{\sqrt{6}-\sqrt{2}}{4}$, and $BD^2 = 4^2 + 7^2 - 4 \cdot 7 \cdot \cos 180 - 75 = 16 + 49 + 28 \cdot \frac{\sqrt{6}-\sqrt{2}}{4}$. So, $AC^2 + BD^2 = 2(16 + 49) = 130$.

13 Let $f(x)$ be the 1000-degree polynomial with all real roots. Given that f has five distinct double roots, twelve distinct triple roots, and no other multiple roots, how many times does $f(x)$ intersect the x-axis?

Answer: $\boxed{971}$

Solution: Each double root intersects the x-axis twice, so the 5 double roots intersect the x-axis 5 times instead of 10. Similarly, each triple root intersects the x-axis thrice, so the 12 triple roots intersect the x-axis 12 times instead of 36. So, $f(x)$ intersects the x-axis $1000 - 5 - 24 = 971$ times.

14 Niklas Khil is pretty good at squaring stuff. He knows $(x + y)^2 = 4$, $(x + z)^2 = 5$, $(z + y)^2 = 9$, and$(x + y + z)^2 = 3/2 - \sqrt{5}/2$. He can do it all. Now adding three numbers? He's not so good at that. Help Niklas find what $x^2 + y^2 + z^2$ is equal to.

Answer: $\boxed{\dfrac{33 + \sqrt{5}}{2}}$

Solution: We have $x^2 + 2xy + y^2 = 4$, $x^2 + 2xz + z^2 = 5$, $y^2 + 2yz + z^2 = 9$, and $x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = \frac{3-\sqrt{5}}{2}$.

So, $x^2 + y^2 + z^2 = (x^2 + 2xy + y^2) + (x^2 + 2xz + z^2) + (y^2 + 2yz + z^2) - (x^2 + y^2 + z^2 + 2xy + 2xz + 2yz)$$= 4 + 5 + 9 - \frac{3-\sqrt{5}}{2} = \frac{33+\sqrt{5}}{2}$.

15 Find positive integer x such that

$$45^3 + 54^3 = x^5 - 3^5$$

Answer: $\boxed{12}$

Solution: $45^3 + 54^3 = 9^3(5^3 + 6^3) = 9^3(5 + 6)(5^2 - 5 \cdot 6 + 6^2) = 9^3(11)(31)$.

Adding 3^5 on both sides, we have

$$\begin{aligned} x^5 &= 3^5 + 3^5(x \cdot 11 \cdot 31) = 3^5(1 + 33 \cdot 31) \\ &= 3^5(1 + (32 + 1)(32 - 1)) \\ &= 3^5 \cdot 32^2 \\ &= 3^5 \cdot 4^5 \end{aligned}$$

So, $x = 3 \cdot 4 = 12$

16 You are making a circular necklace for your friend's birthday. You have 5 identical blue beads and 8 identical green beads. How many different necklaces can you make using all of these beads so that blue beads are not adjacent to each other? A necklace that is the same when flipped over is considered the same necklace.

Answer: $\boxed{5}$

Solution: We have 5 blue beads, and let the gaps between adjacent blue beads be A, B, C, D, and E in that order. Each gap must contain at least one bead, so we now need to distribute the remaining 3 beads among these 5 gaps.

If all three beads end up in the same gap, there's one possible necklace.

If two beads end up in one gap and one bead ends up in another gap, without loss of generality, A gets the additional two beads. Then, the last bead can be in B or C, since D is symmetric to C through reflection and E is symmetric to B through reflection. So, there are two possible necklaces.

If all three beads end up in different gaps, without loss of generality, A gets an additional bead. Then, the other two beads can be in B and C, or in B and D, as the other cases are symmetric through either reflection or rotation.

So, there are 5 possible necklaces.

17 $(2024^2 + 2025) \cdot (2024^2 - 2023)$ can be written as $\frac{x^6-1}{x^2-1}$ for some positive real number x. What is $|x|$?

Answer: $\boxed{2024}$

Solution: We are going to use the sum and difference of cubes to tackle this problem.

$$x^2+x+1=\frac{x^3-1}{x-1} \text{ and } x^2-x+1=\frac{x^3+1}{x+1}$$

Applying the sum and difference of cubes to this equation, we get that $(2024^2+2025) = (2024^2+2024+1) = \frac{2024^3-1}{2024-1}$ and $(2024^2-2023) = (2024^2-2024+1) = \frac{2024^3+1}{2024+1}$

Multiplying these values together, we get:

$$\begin{aligned}(2024^2+2025)\cdot(2024^2-2023) &= \frac{2024^3-1}{2024-1}\cdot\frac{2024^3+1}{2024+1}\\ &= \frac{(2024^3-1)(2024^3+1)}{(2024-1)(2024+1)}\\ &= \frac{2024^6+2024^3-2024^3-1}{2024^2+2024-2024-1}\\ &= \frac{2024^6-1}{2024^2-1}\end{aligned}$$

Therefore, $x=2024$.

18 How many triangles with positive area and integer side lengths less than 30 can be formed such that the side lengths form an increasing geometric sequence?

Answer: $\boxed{5}$

Solution: Let the side lengths be a, ar, and ar^2, such that $r>1$. By the triangle inequality, $a+ar>ar^2$, giving us $r^2-r-1<0$, so $r<\frac{\sqrt{5}+1}{2}$.

Now, we test values of r.

If $r=\frac{3}{2}$, we get $(4,6,9)$, $(8,12,18)$, and $(12,18,27)$.

If $r=\frac{4}{3}$, we get $(9,12,16)$.

If $r=\frac{5}{4}$, we get $(16,20,25)$.

If r has a denominator greater than 4, the largest side length of the triangle will be more than 30. So, there are 5 possible triangles.

19 Bradley rolls three 6-sided dice and records the three numbers. What is the probability that there is a non-degenerate triangle with these three side lengths?

Answer: $\boxed{\frac{37}{72}}$

Solution: If the shortest side is 1, the other two sides must be equal. If the other two sides are 1, there is one triangle, and if the other two dies are not 1, there are 3 triangles. So, there are $1+5\cdot 3=16$ possibilities in this case.

If the shortest side is 2, if the second side is 2 there are 4 possibilities: (2, 2, 2) and (2, 2, 3) and permutations of it. If the second side is 3, 4, or 5, there are 9 possibilities each: (2, 3, 3) and permutations and (2, 3, 4) and permutations. Finally, if the second side is 6, there are 3 possibilities. So, there are $4+3\cdot 9+3 = 34$ total possibilities in this case.

If the shortest side is 3, if the second side is 3 the longest side can be 3, 4, or 5, giving us 7 possibilities. If the second side is 4, the longest side can be 4, 5, or 6, giving us 15 possibilities. If the second side is 5, the longest side can be 5 or 6, for 9 possibilities, if the second side is 6, there are 3 possibilities. So, there are $7 + 15 + 9 + 3 = 34$ possibilities in this case.

If the shortest side is 4, if the second side is 4 the longest side can be 4, 5, or 6, giving us 7 possibilities. If the second side is 5 the longest side can be 5 or 6, giving us 9 possibilities. Finally, if the second side is 6 the longest side must be 6, giving us 3 possibilities. So, there are $7 + 9 + 3 = 19$ possibilities in this case.

If the shortest side is 5, we have $(5, 5, 5), (5, 5, 6)$, and $(5, 6, 6)$, and their permutations, giving us 7 possibilities.

If the shortest side is 6, all sides of our triangle must be 6, giving us 1 possibility.

So, there are a total of $16 + 34 + 34 + 19 + 7 + 1 = 111$ possible triangles, and $6^3 = 216$ ways to role three sides, so the probability there is a non-degenerate triangle is $\frac{111}{216} = \frac{37}{72}$.

20 Wanda the Witch really hates kids. She especially despises them during Halloween, when their audacity and blatant disregard go through the roof. Wanda devises an evil plan to rid her of kids: she builds a magical entry with the illusion of a tray of candy, but unfortunate victims leave only with celery and disappointment. However, Wanda's magic sometimes malfunctions, so that 50% of the time children receive candy. From Wanda's experience, rumors scare kids away if more than 5 kids in a row have their hopes dashed with vegetables. what is the expected number of kids that Wanda will trick before no more kids ring her doorbell?

Answer: $\boxed{63}$

Solution: Let E_n be the expected number of additional kids Wanda tricks when she has tricked n in a row. For all $i > 5$, $E_i = 0$. When she has tricked n in a row, there is a $\frac{1}{2}$ chance she tricks the next kid, so she would have tricked $n+1$ in a row, and a $\frac{1}{2}$ chance she does not trick the next kid, so she would have tricked 0 in a row. Therefore, $E_n = \frac{1}{2}(E_{n+1} + 1) + \frac{1}{2}E_0$. We can calculate

$$E_5 = \frac{1}{2} + \frac{1}{2}E_0$$

$$E_4 = \frac{1}{2}\left(\frac{1}{2}E_0 + \frac{1}{2} + 1\right) + \frac{1}{2}E_0 = \frac{3}{4}E_0 + \frac{3}{4}$$
$$E_3 = \frac{1}{2}\left(\frac{3}{4}E_0 + \frac{3}{4} + 1\right) + \frac{1}{2}E_0 = \frac{7}{8}E_0 + \frac{7}{8}$$
$$E_2 = \frac{15}{16}E_0 + \frac{15}{16}$$
$$E_1 = \frac{31}{32}E_0 + \frac{31}{32}$$
$$E_0 = \frac{63}{64}E_0 + \frac{63}{64}$$

Therefore, $E_0 = 63$.

21 On the number line, a man initially stands on the origin. Each second, if he is currently on k where $k < 2024$, he can move to any positive integer n where $k < n \leq 2023$ with a 2^{k-n} chance and can move to 2024 with a 2^{k-2023} chance. When he reaches 2024, he stops moving. What is the probability that he ever is on 2017?

Answer: $\boxed{\frac{1}{2}}$

Solution: Let P_n be the probability that the man will reach 2017 given that he's standing on n. Clearly, $P_{2017} = 1$ and $P_i = 0$ for all $i > 2017$. We also know

$$P_n = \frac{1}{2}P_{n+1} + \frac{1}{4}P_{n+2} + \frac{1}{8}P_{n+3} + \dots.$$

We can compute $P_{2016} = \frac{1}{2}$ and $P_{2015} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot 1 = \frac{1}{2}$.
Claim: $P_{2017-k} = \frac{1}{2}$ for all $k > 0$.
Proof: We can prove this with induction. Assume this is true for some value of k. Then,

$$P_{2017-k-1} = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^k}\right) + 1 \cdot \frac{1}{2^{k+1}}$$
$$= \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{k+1}} + \frac{1}{2^{k+1}} = \frac{1}{2}$$

Therefore, $P_0 = \frac{1}{2}$.

22 How many integers n are there, where $1 \leq n \leq 2024$ and $n^{n!}$ has a ones digit of 1?

Answer: $\boxed{809}$

Solution: Since $n^{n!}$ ends in 1, n must end in 1, 3, 7, or 9. If n ends in 1, $n^{n!}$ must end in 1, so there are 203 such values of n. If n ends in 3, $n!$ must be a multiple of 4, so $n \geq 4$. There are 203 values of n that end in 3, but $n = 3$ doesn't work, so there are 202 such values of n. If n ends in 7, $n!$ must be a multiple of 4, so $n \geq 4$. There are 202 such values of n. If n ends in 9, $n!$ must be even, so $n \geq 2$. There are 202 such values of n. Therefore, there are a total of 809 values of n.

23 A frog is on a number line. It starts at 0, and each minute it has an equally likely chance to jump to any integer between $c+1$ and n inclusive, where c is the number it currently sits on and n is an arbitrary constant positive number. Let E be the expected number of minutes until the frog lands on n. Find the minimum n such that $E > 3$.

Answer: $\boxed{11}$

Solution: After a jump, the frog is expected to land on the average value of the integers between $c+1$ and n inclusive. So, after the first jump, the frog is expected to land on $\frac{n+1}{2}$, after the second jump, the frog is expected to land on $\frac{\frac{n+1}{2}+1+n}{2} = \frac{3n+3}{4}$, and after the third jump, the frog is expected to land on $\frac{\frac{3n+3}{4}+1+n}{2} = \frac{7n+7}{8}$. After the frog's second jump, we want it to land on $n-2$, so on the third jump, it will land between $n-1$ and n, instead of n. Solving $\frac{3n+3}{4} = n-2$, we get $n = 11$.

24 A circle with diameter AC is intersected by a secant at points B and D. The secant and the diameter intersect at point P outside the circle. Perpendiculars AE and CF are drawn from the extremities of the diameter to the secant. If $EB = 2$ and $BD = 6$, find DF.

Answer: $\boxed{2}$

Solution: Let AE intersect the circle again at X, and let CF intersect the circle again at Y. We know $AE \parallel CF$, and since AC is a diameter, we know $\angle AXC = \angle AYC = 90°$. Therefore, $AEFY$ and $AECY$ are both rectangles. Since AY and BD are parallel chords of the circle, they have the same perpendicular bisector. Rectangle $AEFY$ is symmetric about the perpendicular bisector of AY, so it must be symmetric about the perpendicular bisector of BD, so $DF = EB = 2$.

25 Evaluate

$$\sum_{i=1}^{\infty} \left(\frac{F_n}{10^n}\right)$$

where F_n denotes the nth fibonacci number.

Answer: $\boxed{\frac{10}{89}}$

Solution: Let S denote the sum of the expression. Then,

$$S = \frac{1}{10} + \frac{1}{10^2} + \frac{2}{10^3} + \frac{3}{10^4} + \dots$$

$$\frac{1}{10}S = \frac{1}{10^2} + \frac{1}{10^3} + \frac{2}{10^4} + \frac{3}{10^4} + \dots$$

$$\frac{9}{10}S = S - \frac{1}{10}S = \frac{1}{10} + \left(\frac{1}{10^3} + \frac{1}{10^4} + \dots\right)$$

Therefore, we have $\frac{9}{10}S = \frac{1}{10} + \frac{1}{100}S$, so $S = \frac{10}{89}$.

26 What is the sum of the reciprocals of every non-zero correct answer across all problems appearing on either division of this year's MBMT given that the answer is positive? (This includes all 8 individual rounds, both team rounds, and both guts rounds.)

Answer: $\boxed{43.73836968584040}$

Solution: N/A

27 Estimate $\log_{10}(2024!)$.

Answer: $\boxed{5814.81025022866142}$

Solution: N/A

28 A circle of radius 1 is circumscribed by an equilateral triangle which is circumscribed by another circle which is circumscribed by a square. This pattern continues of circumscribing circles and then regular n-gons up until n = 8. Find the sum of the areas of the even n-gons minus the sum of the areas of the odd n-gons.

Answer: $\boxed{71.34768575207762}$

Solution: N/A

29 We've simulated 1,000 3D random walks, each consisting of ten steps of length one in a random direction. Estimate the total sum of the distances from the origin.

Answer: $\boxed{2853.954988179679}$

Solution: N/A

30 Approximate $\binom{100}{0} + \binom{99}{1} + \ldots + \binom{50}{50}$

Answer: $\boxed{354224848179261915075}$

Solution: N/A