MBMT Gödel Guts Round — Set 1

April 16, 2023

[start=(1)*5+1]

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MBMT Gödel Guts Round − Set 1
April 16, 2023

1 **[3]** If the Earth really was flat and a circle with radius 4 miles, and 75% of the surface area was ocean, what is the surface area NOT covered by ocean in square miles?

2 **[3]** Maggie wants to paint her bedroom blue-gray. To make blue-gray paint, she must mix one part blue paint with two parts white paint. If she has 12 liters of white paint, how much blue-gray paint will she have after she mixes in the right amount of blue paint?

3 **[3]** Let t be the answer to this question. If $x^2 + ax + b$ has exactly one real solution, t, and $a \neq b$, find the value of b.

4 **[3]** Bob has b roses. Bobeth has double the number of roses that Bob has. Boberta has $\frac{1}{4}b^2$ more roses than Bob. How many roses do all three of them have together if Bobeth and Boberta have the same number of roses? They all have at least one rose.

5 **[3]** Bonnie likes to collect stickers. On her birthday, she collects the same number of stickers as the age she is turning. While she was not given any before she was 5, she started by collecting 5 stickers at her 5th birthday. How many stickers will she have in total when she turns 24, including those she collects at her 24th birthday?

MBMT Gödel Guts Round — Set 2

April 16, 2023

[start=(2)*5+1]

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MBMT Gödel Guts Round — Set 2
April 16, 2023

6 **[4]** How many numbers less than or equal to 75 are divisible by 2, 3, or 7?

7 **[4]** Find the area of the largest equilateral triangle that can be inscribed in a unit square.

8 **[4]** Taylor drops a surprise album from a height of 13 meters. If the height of each bounce is 2/3 the height of the previous bounce, find the total vertical distance traveled by the album.

9 **[4]** Bob can do 5 pages of math homework in 3 hours. Jeff can do 5 pages of math homework in 4 hours. If Bob and Jeff worked together, how long would it take them to finish 35 pages of math homework?

10 **[4]** A "crescent" is made by putting a smaller circle inside a larger circle (shown below). The smaller circle's radius is 2/3 of the larger circle's radius. If the larger circle has radius 9, what is the area of the crescent (shaded region)?

MBMT Gödel Guts Round — Set 3

April 16, 2023

[start=(3)*5+1]

MBMT Gödel Guts Round — Set 3
April 16, 2023

11 **[5]** How many ways are there to tile a 99x102 grid with octominoes the shape of 3x3 blocks with a single missing corner? A grid is considered tiled if it is covered with no gaps.

12 **[5]** In a parallelogram ABCD, AB = 4, BC = 7, and ∠ ABC = 75°. Find AC^2+ BD^2.

13 **[5]** Let $f(x)$ be the 1000-degree polynomial with all real roots. Given that f has five distinct double roots, twelve distinct triple roots, and no other multiple roots, how many times does $f(x)$ intersect the x-axis?

14 **[5]** Niklas Khil is pretty good at squaring stuff. He knows $(x + y)^2 = 4$, $(x + z)^2 = 5$, $(z + y)^2 = 9$, and $(x + y + z)^2 = 3/2 - \sqrt{5}/2$. He can do it all. Now adding three numbers? He's not so good at that. Help Niklas find what $x^2 + y^2 + z^2$ is equal to.

15 **[5]** Find positive integer x such that

$$45^3 + 54^3 = x^5 - 3^5$$

MBMT Gödel Guts Round — Set 4

April 16, 2023

[start=(4)*5+1]

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16 **[7]** You are making a circular necklace for your friend's birthday. You have 5 identical blue beads and 8 identical green beads. How many different necklaces can you make using all of these beads so that blue beads are not adjacent to each other? A necklace that is the same when flipped over is considered the same necklace.

17 **[7]** $(2024^2 + 2025) \cdot (2024^2 - 2023)$ can be written as $\frac{x^6 - 1}{x^2 - 1}$ for some positive real number x. What is $|x|$?

18 **[7]** How many triangles with positive area and integer side lengths less than 30 can be formed such that the side lengths form an increasing geometric sequence?

19 **[7]** Bradley rolls three 6-sided dice and records the three numbers. What is the probability that there is a non-degenerate triangle with these three side lengths?

20 **[7]** Wanda the Witch really hates kids. She especially despises them during Halloween, when their audacity and blatant disregard go through the roof. Wanda devises an evil plan to rid her of kids: she builds a magical entry with the illusion of a tray of candy, but unfortunate victims leave only with celery and disappointment. However, Wanda's magic sometimes malfunctions, so that 50% of the time children receive candy. From Wanda's experience, rumors scare kids away if more than 5 kids in a row have their hopes dashed with vegetables. what is the expected number of kids that Wanda will trick before no more kids ring her doorbell?

MBMT Gödel Guts Round — Set 5

April 16, 2023

[start=(5)*5+1]

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21 **[9]** On the number line, a man initially stands on the origin. Each second, if he is currently on k where $k < 2024$, he can move to any positive integer n where $k < n \leq 2023$ with a 2^{k-n} chance and can move to 2024 with a 2^{k-2023} chance. When he reaches 2024, he stops moving. What is the probability that he ever is on 2017?

22 **[9]** How many integers n are there, where $1 \leq n \leq 2024$ and $n^{n!}$ has a ones digit of 1?

23 **[9]** A frog is on a number line. It starts at 0, and each minute it has an equally likely chance to jump to any integer between $c + 1$ and n inclusive, where c is the number it currently sits on and n is an arbitrary constant positive number. Let E be the expected number of minutes until the frog lands on n. Find the minimum n such that $E > 3$.

24 **[9]** A circle with diameter AC is intersected by a secant at points B and D. The secant and the diameter intersect at point P outside the circle. Perpendiculars AE and CF are drawn from the extremities of the diameter to the secant. If $EB = 2$ and $BD = 6$, find DF.

25 **[9]** Evaluate
$$\sum_{i=1}^{\infty} \left(\frac{F_n}{10^n} \right)$$
where F_n denotes the nth fibonacci number.

MBMT Gödel Guts Round — Set 6

April 16, 2023

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26 **[12]** What is the sum of the reciprocals of every non-zero correct answer across all problems appearing on either division of this year's MBMT given that the answer is positive? (This includes all 8 individual rounds, both team rounds, and both guts rounds.)

27 **[12]** Estimate $\log_{10}(2024!)$.

28 **[12]** A circle of radius 1 is circumscribed by an equilateral triangle which is circumscribed by another circle which is circumscribed by a square. This pattern continues of circumscribing circles and then regular n-gons up until n = 8. Find the sum of the areas of the even n-gons minus the sum of the areas of the odd n-gons.

29 **[12]** We've simulated 1,000 3D random walks, each consisting of ten steps of length one in a random direction. Estimate the total sum of the distances from the origin.

30 **[12]** Approximate $\binom{100}{0} + \binom{99}{1} + \ldots + \binom{50}{50}$