1 Betty travels 6 parsecs north and 8 parsecs west to get to school. August leaves from the same location as Betty except he travels in a straight line directly to school. How much more distance did Betty travel than August?

Answer: $\boxed{4}$

Solution: Betty travels along the legs of a right triangle with legs of length 6 and 8. Betty travels a total distance of $6 + 8 = 14$. From the Pythagorean theorem, August travels a distance of $\sqrt{6^2 + 8^2} = 10$. Thus, Betty travels $14 - 10 = 4$ more parsecs than August.

2 5 congruent squares with side length 2 are packaged together below. The middle square is offset by 45 degrees and is tangent to each of the other squares at the midpoint of each of its sides. Find the distance between the marked corners.

Answer: $\boxed{3}$

Solution: Draw a diagonal of the large square; this diagonal runs through two diagonals and a side length of the smaller square. As such, the length of the large diagonal is $2\sqrt{2} \cdot 2 + 2 = 2 + 4\sqrt{2}$. The side length is therefore $4 + \sqrt{2}$. The two legs have lengths $2 + \frac{\sqrt{2}}{2}$ and $2 - \frac{\sqrt{2}}{2}$, so using the Pythagorean Theorem the length between the two vertices has length

$$\sqrt{(2 + \frac{\sqrt{2}}{2})^2 + (2 - \frac{\sqrt{2}}{2})^2} = 3.$$

3 In triangle XYZ, two sides are 5 and 10 units long, and the angle between them is 60 degrees. Find the area of the triangle.

Answer: $\boxed{\dfrac{25\sqrt{3}}{2}}$

Solution: The area of a triangle given two sides and the angle *between* the two sides is $\frac{1}{2}ab\sin\theta$, where a and b are the two side lengths and θ is the angle between. Therefore, the area of XYZ is $A = \frac{1}{2} \cdot 5 \cdot 10 \cdot \sin 60°$. We substitute $\sin 60° = \frac{\sqrt{3}}{2}$ into A and get $A = \frac{1}{2} \cdot 5 \cdot 10 \cdot \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{2}$ square units.

4 Arnold the Ant starts at the top of a regular octahedron with side length 2. What is the shortest distance Arnold needs to walk to reach the opposite corner, given that he can only travel along the surface of the octahedron?

Answer: $\boxed{2\sqrt{3}}$

Solution: Arnold must first reach the perimeter of the horizontal sqaure boundary, then proceed from there to reacht he bottom vertex. The shortest possible distance to the horizontal square boundary is $\sqrt{3}$, the altitude of the face equilateral triangle. The shortest path from the horizontal square boundary to the bottom is also $\sqrt{3}$ and starts at the same point that the top path ends. As such, the shortest total possible distance is simply $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$.

5 What is the area of the region of all points that are of distance at most 1 from $(0,0)$ or $(1,0)$?

Answer: $\boxed{\frac{4\pi}{3}-\frac{\sqrt{3}}{2}}$

Solution: Let $A=(0,0)$ and $B=(1,0)$. Let's call the circle of radius 1 centered at A as $\mathcal{C}_1$ and the circle of radius 1 centered at B as $\mathcal{C}_2$. From drawing a diagram, we see that $\mathcal{C}_1$ and $\mathcal{C}_2$ intersect at some points C and D. By symmetry, C and D should have x-coordinates $\frac{1}{2}$, and are reflected across the x-axis from one another. So, let's let $C=(\frac{1}{2},y)$ and $D=(\frac{1}{2},-y)$ for some positive number y.

We can calculate y by noting that $AC=AD=1$. By the distance formula, that means $1=\sqrt{(\frac{1}{2}-0)^2+(y-0)^2}$, so $\frac{3}{4}=y^2\to y=\frac{\sqrt{3}}{2}$.

To find the area of the region with $\mathcal{C}_1$ and $\mathcal{C}_2$, note that we can find the individual areas of $\mathcal{C}_1$ and $\mathcal{C}_2$ and then subtract the overlap (because it's been counted twice).

The overlap consists of $\Delta ABC, \Delta ABD$, and four little "caps". The area of ΔABC and ΔABD are each $\frac{\sqrt{3}}{4}$, since each is an equilateral triangle with side length 1 and the area of an equilateral triangle with side length s is $\frac{\sqrt{3}}{4}s^2$. Each "cap" is a sixth of a circle with radius 1, with the equilateral triangle of side length 1 subtracted out. Therefore, $A_{\text{cap}}=\frac{1}{6}\cdot\pi 1^2-\frac{\sqrt{3}}{4}=\frac{\pi}{6}-\frac{\sqrt{3}}{4}$. Then, the total overlap is $\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}+4\cdot\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)=\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$. Remember this is the overlap we wanted to subtract from the individual areas of $\mathcal{C}_1$ and $\mathcal{C}_2$, so our final area of the figure is $\pi\cdot 1^2+\pi\cdot 1^2-\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)=\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$.

6 In rectangle ABCD, AB is 3 and BC is 2. Let E lie on side CD, and let circle O be inscribed inside triangle ABE. What is the maximal possible area of O?

Answer: $\boxed{\frac{9\pi}{16}}$

Solution: Notice that the areas of all such triangles ABE are constant and equal to 3, as the base and height are always 3 and 2, respectively. We know that, given the perimeter p and inradius r of the triangle, the area is equal to $\frac{pr}{2}$. This tells us that $r=\frac{6}{p}$, so we wish to minimize p to maximize the inradius. Since $AB=3$, this boils down to trying to minimize $AB+BE$.
Imagine if we reflect point B across point C to a new point B'. By symmetry, $BE=B'E$, so $AE+BE=AE+B'E$. This length is minimized when A, E, and B' are collinear, giving a length of $\sqrt{3^2+4^2}=5$. Then the perimeter is equal to 8, giving an inradius of $\frac{3}{4}$. As such, the area of the incircle is $\frac{9\pi}{16}$.

7 Triangle ABC is inscribed in circle O. Point P is drawn outside circle O such that P, B, and C are collinear in that order, and PA is tangent to the circle. Given that PB and AB are both integers and PB · AB = 10, what is the maximal area of triangle PAC?

Answer: $\boxed{500}$

Solution: We have $PC \cdot PB = PA^2$, so $PC = \frac{PA^2}{PB}$. note that $[PCA] = \frac{PC}{PB}[PBA]$, so $[PCA] = \frac{PA^2}{PB^2}[PBA]$. $[PBA] = \frac{1}{2}PA \cdot PB \cdot \sin\angle BPA$: $PA \cdot PB$ is known to be 10 and the sine is maximized at 1, so $[PBA]$ is maximized at 5. $\frac{PA^2}{PB^2}$ is maximized when PB^2 is minimized, which occurs when $PA = 10$ and $PB = 1$: this gives $[PCA] = 100 * 5 = 500$. If you visualize a diagram this is in fact possible where P is barely off the circle and the circle has a massive radius.

8 Triangle ABC has AB = 5, AC = 6, BC = 7. The circumcenter of the triangle has center O. Segment BO is extended through side AC to meet the circumcircle at point B'. What is the length of B'C?

Answer: $\boxed{\dfrac{7\sqrt{6}}{12}}$

Solution: We know that $\triangle BB'C$ is right and we know BC, so all we need to do is to find the circumradius of ABC. The area of a triangle can be found in two ways. From Heron's Formula, the area is ABC is $\sqrt{9 \cdot 4 \cdot 3 \cdot 2} = 6\sqrt{6}$. Furthermore, the area can be written as $\frac{abc}{4R}$. Solving gives $R = \frac{35}{4\sqrt{6}}$. Finally, using the Pythagorean Theorem gives $B'C = \frac{7\sqrt{6}}{12}$.