MBMT Geometry Round — Gödel

April 16, 2023

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** Betty travels 6 parsecs north and 8 parsecs west to get to school. August leaves from the same location as Betty except he travels in a straight line directly to school. How much more distance did Betty travel than August?

_____ **2** 5 congruent squares with side length 2 are packaged together below. The middle square is offset by 45 degrees and is tangent to each of the other squares at the midpoint of each of its sides. Find the distance between the marked corners.

_____ **3** In triangle XYZ, two sides are 5 and 10 units long, and the angle between them is 60 degrees. Find the area of the triangle.

_____ **4** Arnold the Ant starts at the top of a regular octahedron with side length 2. What is the shortest distance Arnold needs to walk to reach the opposite corner, given that he can only travel along the surface of the octahedron?

_____ **5** What is the area of the region of all points that are of distance at most 1 from $(0, 0)$ or $(1, 0)$?

_____ **6** In rectangle ABCD, AB is 3 and BC is 2. Let E lie on side CD, and let circle O be inscribed inside triangle ABE. What is the maximal possible area of O?

_____ **7** Triangle ABC is inscribed in circle O. Point P is drawn outside circle O such that P, B, and C are collinear in that order, and PA is tangent to the circle. Given that PB and AB are both integers and $PB \cdot AB = 10$, what is the maximal area of triangle PAC?

_____ **8** Triangle ABC has $AB = 5$, $AC = 6$, $BC = 7$. The circumcenter of the triangle has center O. Segment BO is extended through side AC to meet the circumcircle at point B'. What is the length of B'C?