

# MBMT Counting and Probability Round – Gödel

April 16, 2023

Full Name \_\_\_\_\_

Student ID Number \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- \_\_\_\_\_ 1 Jenny bought 30 of mystery boxes and received 6 large prizes. If the probability of winning a large prize stays the same, how many large prizes is she expected to win from 20 mystery boxes?
- \_\_\_\_\_ 2 Every day, Mr. Schwartz has a 30% chance of chucking a ball of paper at a sleeping student. What is the probability that he doesn't throw paper balls for 2 consecutive days? Express your answer as a common fraction.
- \_\_\_\_\_ 3 Jason has a fair coin. Evan has a weighted coin where the probability of landing heads is  $\frac{3}{4}$ . What is the minimum number of flips such that the probability that Evan lands tails for exactly one of his flips is more than twice the probability that Jason lands tails for exactly one of his flips?
- \_\_\_\_\_ 4 Lewis is learning to spell. He writes out all of the arrangements of the letters in his name and orders them alphabetically. For example, the first arrangement would be "eilsw". What position is the word "lewis" in?
- \_\_\_\_\_ 5 Yunyi has 25 pieces of cheese. He wants to split some (perhaps all) of the cheese between him and his two friends such that each person gets at least 5 pieces. In how many ways can this be done?
- \_\_\_\_\_ 6 Nicholas likes words because they make sense. How many rearrangements of the letters in the word "senselessness" contain the word "sense" somewhere within it?
- \_\_\_\_\_ 7 You have a die that can roll any integer between 1 and 20 inclusive with equal probability. You want to design another custom die that can roll any integer between 1 and  $x$  inclusive with equal probability. Which positive integer  $x$  should you choose to maximize the probability that, when you independently roll your two dice, their values sum to 24?
- \_\_\_\_\_ 8 A sequence of 7 digits is "pandemic" if it consists entirely of the digits 0 and 2, and it's possible to remove 3 digits so that the remaining 4 read 2020 in order. For example, 2022200 is pandemic because we can remove the 4th, 5th, 6th digits from the left so that the remaining digits form 2020, but 2200000 is not pandemic. How many pandemic strings are there?