1 Carlos is piloting his drone. Each move, his drone can either move up 4 feet, or move down 2 feet. What is the least number of moves in which Carlos can move his drone up exactly 10 feet from its starting position?

Answer: $\boxed{4}$

Solution: Carlos must move his drone up at least three times to exceed 10 feet. After he does so, moving down once brings him to exactly 10 feet, for a total of 4 moves.

2 Let f be a linear function ($f(x) = ax + b$ for some a and b). If $f(z) = 8$, $f(f(z)) = 20$, and $f(f(f(z))) = 44$, what is the value of z?

Answer: $\boxed{2}$

Solution: By substitution, $f(f(z)) = f(8) = 20$ and $f(f(f(z))) = f(20) = 44$. The linear function that matches $f(8) = 20$ and $f(20) = 44$ is $y = 2x + 4$. To solve for z, $2z + 4 = 8$, so $z = 2$.

3 In an arithmetic sequence a_n, the terms a_1, a_2, and a_5 form a geometric sequence. Given that $a_1 = 2$, what is the largest possible value of a_{10}? (An arithmetic sequence is a list of numbers where the difference between consecutive terms is constant. A geometric sequence is a list of numbers where the ratio between consecutive terms is constant.)

Answer: $\boxed{38}$

Solution: Because the sequence is arithmetic, term $a_2 = 2 + d$ for some common difference d. Likewise, $a_5 = 2 + 4d$. Since a_1, a_2, and a_5 form geometric sequences, it follows that $\frac{a_2}{a_1} = \frac{a_5}{a_2}$, so $\frac{2+d}{2} = \frac{2+4d}{2+d}$. Cross-multiplying, we get the quadratic $d^2 + 4d + 4 = 8d + 4$, or $d^2 - 4d = 0$. Obviously, a_{10} is maximized when $d = 4$, so $a_{10} = 2 + 9 \cdot 4 = 38$.

4 For real a and b, let $f(x) = (2a - x)(x + 3)$. $f(x)$ has a maximum value of $2b$. Furthermore, let $g(x) = bx^2 + 8x - 6$, where $g(x)$ has a vertex with y-coordinate $-b$. Find the maximum value of $a^2 \cdot b$.

Answer: $\boxed{242}$

Solution: We can first rewrite $g(x)$ in vertex form by completing the square: $g(x) = b(x+\frac{4}{b})^2 - 6 - \frac{16}{b}$. Setting the y-coordinate equal to $-b$, we get $-6 - \frac{16}{b} = -b \Rightarrow b^2 - 6b - 16 = 0 \Rightarrow (b+2)(b-8) = 0 \Rightarrow b = -2, 8$. For f(x), the maximum value is the y-coordinate of the vertex, and the vertex's x-coordinate is the midpoint of the x-intercepts. The x-coordinate of the vertex is therefore $\frac{2a-3}{2}$. Plugging this into $f(x)$, we get $(\frac{2a+3}{2})^2 = 2b$. The left hand side is non-negative, so b = 8. $(\frac{2a+3}{2})^2 = 16 \Rightarrow 2a + 3 = \pm 8 \Rightarrow a = \frac{-3\pm 8}{2}$. To maximize $a^2 \cdot b$, $a = -\frac{11}{2}, b = 8$, giving us 242.

5 There are three positive roots to the equation $x^3 - 10x^2 + 26x + d$. If the three roots form the side lengths of a right triangle, find the value of d.

Answer: $\boxed{240 - 100\sqrt{6}}$

Solution: Let the roots be r_1, r_2, and r_3. Note that $r_1^2 + r_2^2 + r_3^2 = (r_1 + r_2 + r_3)^2 - 2(r_1r_2 + r_1r_3 + r_2r_3) = 100 - 52 = 48$ by Vieta's Formula. WLOG, let $r_3 > r_2 > r_1$. Then, r_3 must be the hypotenuse of the triangle, so $r_1^2 + r_2^2 = r_3^2$, meaning $2r_3^2 = 48$, or $r_3 = 2\sqrt{6}$. Applying Vieta's again, $r_1 + r_2 + r_3 = 10$, so $r_1 + r_2 = 10 - 2\sqrt{6}$. Then, $r_1r_2 = \frac{(r_1+r_2)^2-(r_1^2+r_2)^2}{2} = 50 - 20\sqrt{6}$, and multiplying by r_3 we get $-d = r_1r_2r_3 = 100\sqrt{6} - 240$, so d is $240 - 100\sqrt{6}$.

6 The product of the solutions to the equation $\log_2 2x + 8\log_{4x} 2 = 20$ is equal to y. Find $\log_2 y$.

Answer: $\boxed{17}$

Solution: Using log identities, we can rewrite the equation into $\log_2 x + log_2 2 + 8\frac{log_2 2}{\log 24x} = 20$. Simplifying and using change of base, we get $\log_2 x + 8\frac{1}{\log 2x+2} = 20$. Setting $n = \log_2 x$, and rearranging the equation we get $n^2 - 17n - 30 = 0$. Letting the solutions to this equation be p and q whose corresponding x values are x_1 and x_2, $p + q = \log 2x_1 \cdot x_2 \Rightarrow y = x_1 \cdot x_2 = 2^{p+q}$. By Vieta's, the sum of the solutions to the quadratic is 17. Therefore, $\log 2y = log_2 2^{p+q} = 17$.

7 The floor function, denoted as $\lfloor x \rfloor$, outputs the greatest integer less than or equal to x. Find the area of the region above the x-axis and below the function $x\lfloor\sqrt{x}\rfloor$ between $x = 1$ and $x = 25$.

Answer: $\boxed{1073}$

Solution: On the interval $[1, 4)$, $y = x\lfloor\sqrt{x}\rfloor$ is a segment from $(1, 1)$ to $(4, 4)$, so the area in this interval is $\frac{3\cdot(1+4)}{2} = \frac{15}{2}$. On the interval $[4, 9)$, $y = x\lfloor\sqrt{x}\rfloor$ is a segment from $(4, 8)$ to $(9, 18)$, so the area in this interval is $\frac{5\cdot(8+18)}{2} = 65$. On the interval $[9, 16)$, $y = x\lfloor\sqrt{x}\rfloor$ is a segment from $(9, 27)$ to $(16, 48)$, so the area in this interval is $\frac{7\cdot(27+48)}{2} = \frac{525}{2}$. On the interval $[16, 25)$, $y = x\lfloor\sqrt{x}\rfloor$ is a segment from $(16, 64)$ to $(25, 100)$, so the area in this interval is $\frac{9\cdot(64+100)}{2} = 738$. So, the total area is $\frac{15}{2} + 65 + \frac{525}{2} + 738 = 1073$.

8 There exists an angle x between 0 and $\frac{\pi}{2}$ exclusive, such that $4\sin(2x) + 4\sin(x) + 12\sin^2\left(\frac{x}{2}\right) = 9$. Find $\cos(x)$.

Answer: $\boxed{\frac{\sqrt{7}}{4}}$

Solution: We can rewrite the equation using the double and half angle identities: $\sin 2x = 2\sin x\cos x$ and $\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$.

$$4\sin 2x + 4\sin x + 12\sin^2\frac{x}{2} = 9$$

$$8\sin x\cos x + 4\sin x + 12\left(\frac{1-\cos x}{2}\right) = 9$$

$$8\sin x\cos x + 4\sin x - 6\cos x - 3 = 0$$

$$4\sin x(2\cos x + 1) - 3(2\cos x + 1) = 0$$

$$(4\sin x - 3)(2\cos x + 1) = 0$$
$$\sin x = \frac{3}{4}, \cos x = -\frac{1}{2}$$

There is only one x that satisfies this for which $\cos x = \frac{\sqrt{7}}{4}$.