

MBMT Algebra Round – Gödel

April 16, 2023

Full Name _____

Student ID Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- _____ 1 Carlos is piloting his drone. Each move, his drone can either move up 4 feet, or move down 2 feet. What is the least number of moves in which Carlos can move his drone up exactly 10 feet from its starting position?
- _____ 2 Let f be a linear function ($f(x) = ax + b$ for some a and b). If $f(z) = 8$, $f(f(z)) = 20$, and $f(f(f(z))) = 44$, what is the value of z ?
- _____ 3 In an arithmetic sequence a_n , the terms a_1 , a_2 , and a_5 form a geometric sequence. Given that $a_1 = 2$, what is the largest possible value of a_{10} ? (An arithmetic sequence is a list of numbers where the difference between consecutive terms is constant. A geometric sequence is a list of numbers where the ratio between consecutive terms is constant.)
- _____ 4 For real a and b , let $f(x) = (2a - x)(x + 3)$. $f(x)$ has a maximum value of $2b$. Furthermore, let $g(x) = bx^2 + 8x - 6$, where $g(x)$ has a vertex with y-coordinate $-b$. Find the maximum value of $a^2 \cdot b$.
- _____ 5 There are three positive roots to the equation $x^3 - 10x^2 + 26x + d$. If the three roots form the side lengths of a right triangle, find the value of d .
- _____ 6 The product of the solutions to the equation $\log_2 2x + 8 \log_{4x} 2 = 20$ is equal to y . Find $\log_2 y$.
- _____ 7 The floor function, denoted as $\lfloor x \rfloor$, outputs the greatest integer less than or equal to x . Find the area of the region above the x -axis and below the function $x \lfloor \sqrt{x} \rfloor$ between $x = 1$ and $x = 25$.
- _____ 8 There exists an angle x between 0 and $\frac{\pi}{2}$ exclusive, such that $4 \sin(2x) + 4 \sin(x) + 12 \sin^2\left(\frac{x}{2}\right) = 9$. Find $\cos(x)$.