1 Evan calculates the mean of the set 60, 63, 68, 71, 73. However, he mistakenly puts 70 into his calculator instead of 60. What is the absolute difference between the mean of the set and the incorrect mean Evan calculates?

Answer: $\boxed{2}$

Solution: The total sum of the set is $\text{sum} = 60+63+68+71+73$. Evan's error results in $70+60+68+71+73$, resulting in an increase of the sum by 10. Since $\text{mean} = \frac{\text{sum}}{5}$, the absolute difference is $\frac{\text{sum}+10}{5} - \frac{\text{sum}}{5} = \frac{10}{5} = 2$.

2 A regular pentagon (5 sides) and a regular nonagon (9 sides) have the same perimeter. If the side length of the pentagon is 18, what is the side length of the nonagon?

Answer: $\boxed{10}$

Solution: The total perimeter is $5 \cdot 18 = 90$. Since a nonagon has 9 sides, the side length of the nonagon is $90 \div 9 = 10$.

3 Ada is collecting pesetas at a rate of 5 per minute. If Ada started out with a certain amount of pesetas, and after 18 minutes, she has quadrupled the amount of pesetas she has, how many pesetas does Ada have after one hour?

Answer: $\boxed{330}$

Solution: At some time t minutes from the start, $\text{Total Pesetas} = 5 \cdot t + c$, where c represents the initial number of Pesetas. Then, $4 \cdot c = 5 \cdot 18 + c$ which implies $3c = 90$. Thus, $c = 30$. At time $t = 60$, $\text{Total Pesetas} = 5 \cdot 60 + 30 = 330$.

4 Anne wants to crochet a rectangular blanket with 36 square meters of yarn. If the side lengths of her blanket must be positive integers and she uses up all of her yarn in the process - how many different widths can her blanket have?

Answer: $\boxed{9}$

Solution: This problem is equivalent to finding how many integer pairs divide 36. The prime factorization of 36 is $2^2 \cdot 3^2$. This gives it $(2+1) \cdot (2+1)$ different factors, giving our answer of 9.

5 Timmy is playing a game where he is given an integer x amount of money between 1 and 100 inclusive, but then has a $x\%$ chance of losing the money. What amount of money should Timmy choose to maximize his profit?

Answer: $\boxed{50}$

Solution: This is equivalent to maximizing expected value. The expected amount of money earned is $0 \cdot \frac{x}{100} + x \cdot \frac{100-x}{100} = x\frac{100-x}{100} = -\frac{1}{100} \cdot (x^2 - 100x)$. The maximum of a parabola is at the vertex. Therefore, by completing the square we get $-\frac{1}{100}(x-50)^2 + 25$, which as a vertex at (50,25). Thus, Timmy should choose 50.

6 How many 2-digit primes have digits summing to a prime?

Answer: $\boxed{10}$

Solution: Note that all primes above 2 are odd. Since the tens digit and ones digit must sum to a prime, and all primes greater than two are odd, the tens digit must be even. However, we must also consider summing to two, which reveals that 11 works. Finally, by bashing all possible combinations of 20, 40, 60, 80, and 1, 3, 7, 9, we find the solutions are $23, 29, 41, 43, 47, 61, 67, 83, 89$. Thus, our answer is 10.

7 Paula has a regular hexagon with a side length of 3. She draws an outward-facing equilateral triangle on each side and connects the outer vertices to create a larger hexagon. What is the side length of the larger hexagon?

Answer: $\boxed{3\sqrt{3}}$

Solution: The isosceles triangle formed on the outside has angles of 120, 30, and 30. Then the side length of the larger hexagon is simply $\frac{3\sqrt{3}}{2} \cdot 2 = 3\sqrt{3}$.

8 If $x^2 + y^2 - 6x - 2y + 10 = 0$, find $3x + 2y$.

Answer: $\boxed{11}$

Solution: By completing the square, we get $(x-3)^2 + (y-1)^2 = 0$. Each term must be 0, giving $x = 3$ and $y = 1$. Plugging in $(3, 1)$ gives us an answer of 11.

9 How many 4-digit integers have digits multiplying to 24?

Answer: $\boxed{64}$

Solution: The possible (unordered) lists of numbers that can be digits are $(1, 1, 4, 6)$, $(1, 1, 3, 8)$, $(1, 2, 2, 6)$, $(2, 2, 2, 3)$, and $(1, 2, 3, 4)$. In total the number of four digit integers is $12 + 12 + 12 + 4 + 24 = 64$.

10 In right triangle △ ABC, angle C is 90 degrees and AC = 10. If angle A is between 45 and 60 degrees, exclusive, how many integer values could side length BC possibly be?

Answer: $\boxed{7}$

Solution: When angle A is 45 degrees $BC = 10$. When A is 60 degrees BC is $10\sqrt{3}$, which is approximately 17.3. In total we have 7 integers between 10 and 17.3 exclusive.

11 The circle $(x-5)^2 + (y-3)^2 = 16$ is tangent to the line $y = x + b$. Find the sum of all possible values of b.

Answer: $\boxed{-4}$

Solution: The first point of tangency has coordinates $(5 - 2\sqrt{2}, 3 + 2\sqrt{2})$, from which we can find the first y-intercept (using 45-45-90 triangles) to be $4\sqrt{2} - 2$. Drawing a trapezoid using the two tangent lines, we can find the total length between the y-intercepts to be $8\sqrt{2}$. As such, the second y-intercept is at $-4\sqrt{2} - 2$. Summing these gives -4.

12 What is the number of 9-move paths from (0, 0) to (3, 4) where each move has length 1 and can be up, left, down, or right?

Answer: $\boxed{1134}$

Solution: Let U be a move up, D be a move down, R be a move right, and L be a move left. Then the number of U's must be 4 more than the number of D's and the number of R's must be 3 more than the number of L's. As such we can have either 4 U's, 0 D's, 4 R's, and 1 L or 5 U's, 1 D, 3 R's, and 0 L's. Accounting for order, the total number of moves is $\binom{9}{4}\binom{5}{4} + \binom{9}{5}\binom{4}{3} = 630 + 504 = 1134$.

13 How many lattice points lie on the interior of the figure described by the equation $(|2x| + |3y| - 12)^2 < 36$?

Answer: $\boxed{88}$

Solution: Taking the square root and adding 12, we get $6 < |2x| + |3y| < 18$. Graphing gives us two concentric rhombuses, with the inner one having diagonals of length 6 and 4 and the outer one having diagonals of length 18 and 12. We can just count one quadrant, multiply by 4, and add the lattice points on the axes. Looking in quadrant one, the total number of lattice points is $4 + 4 + 3 + 3 + 2 + 1 + 1 = 18$. The number of lattice points on the axes is $2 \cdot (3 + 5) = 16$. As such, the total number of lattice points is $72 + 16 = 88$.

14 Find the smallest positive integer k such that $k \cdot (2^2 - 1)(3^2 - 1)(4^2 - 1)...(2024^2 - 1)$ is a perfect square.

Answer: $\boxed{253}$

Solution: Since $n^2 - 1 = (n - 1)(n + 1)$, we can rewrite the expression as

$$k(2-1)(2+1)(3-1)(3+1)\dots(2024-1)(2024+1)$$
$$= k \cdot 1 \cdot 2 \cdot 3^2 \cdot 4^2 \cdot \dots \cdot 2023^2 \cdot 2024 \cdot 2025$$

Since 2025 is already a perfect square, we need $k \cdot 2 \cdot 2024$ to be a perfect square. We can factor this into $k \cdot 2^4 \cdot 11 \cdot 23$, so k must have a factor of 11 and of 23. The smallest possible value of k is $11 \cdot 23 = 253$.

15 Define an operation $x \;\&\; y = x + y^2$. What is $((...(((1 \;\&\; 3) \;\&\; 5) \;\&\; ...) \;\&\; 19)$?

Answer: $\boxed{1330}$

Solution: Note that $(x \;\&\; y) \;\&\; z = (x + y^2) \;\&\; z = x + y^2 + z^2$. Therefore,

$$((...(((1 \;\&\; 3) \;\&\; 5) \;\&\; ...) \;\&\; 19) = 1 + 3^2 + 5^2 + \dots + 19^2$$
$$= \left(1^2 + 2^2 + 3^2 + \dots + 19^2\right) - \left(2^2 + 4^2 + \dots + 18^2\right)$$
$$= \left(1^2 + 2^2 + 3^2 + \dots + 19^2\right) - 4\left(1^2 + 2^2 + \dots + 9^2\right)$$
$$\frac{19 \cdot 20 \cdot 39}{6} - 4 \cdot \frac{9 \cdot 10 \cdot 19}{6}$$
$$= 1330$$