**1** A car is falling at a constant rate of 5 meters per second. A bus is falling at a constant rate of 7.5 meters per second. If they start at the same height, what is the difference in their heights in 1 minute? There are 60 seconds in a minute.

**Answer:** $\boxed{150}$

**Solution:** In the one minute, the car falls

$$\frac{5\text{meters}}{\text{second}} \cdot 60\text{second} = 300\text{meters}$$

and the bus falls

$$\frac{7.5\text{meters}}{\text{second}} \cdot 60\text{second} = 450\text{meters}.$$

Since they start at the same height, the different in their final heights is $450 - 300 = 150$ meters.

**2** The seniors at Montgomery Blair High School are going on a field trip. There will be 200 students and 25 teachers on the trip. Each bus can carry 45 passengers. How many buses will be needed?

**Answer:** $\boxed{5}$

**Solution:** There are a total of 200+25=225 people, and since each bus has 45 people, we need $\frac{225}{45} = 5$ buses.

**3** The sum of two primes is 31. What is the value of the larger prime? A prime number is a number whose only divisors are one and itself (one is not a prime).

**Answer:** $\boxed{29}$

**Solution:** Since the sum of the two numbers is odd, one number must be odd and the other must be even. 2 is the only even prime, so one number must be 2. This makes the other number 29, which is prime.

**4** A number is called relatively prime to another number if they share no factors other than 1. How many positive integers less than 23 are relatively prime to 23?

**Answer:** $\boxed{22}$

**Solution:** Since 23 is prime, all numbers that are not multiples of 23 and relatively prime to 23. Furthermore, no positive integer less than 23 can be a multiple of 23, so all positive integers less than 23 are relatively prime to 23. There are 22 positive integers less than 23.

**5**  Find the number of divisors of 2024.

**Answer:** $\boxed{16}$

**Solution:**  We can prime factorize 2024 into $2^3 \cdot 11 \cdot 23$. Let $d$ be any divisor of 2024. Then, $d$ must be of the form $2^a \cdot 11^b \cdot 23^c$, and furthermore, $0 \le a \le 3$, $0 \le b \le 1$, and $0 \le c \le 1$. Therefore, we have 4 choices for $a$ and 2 choices each for $b$ and $c$, for a total of 16 possible values of $d$.

**6**  If $x, y$ are nonnegative integers, and $xy + x + 3y = 1$, find $x + y$.

**Answer:** $\boxed{1}$

**Solution:**   If $y \ge 1$, then $xy \ge 0$ and $3y \ge 3$, so $xy + x + 3y \ge 3 \neq 1$. Therefore, $y < 1$, so $y = 0$. This gives us $x = 1$, so $x + y = 1$.

**7**  Manny likes mung beans. He can purchase 20 beans for \$3, 50 beans for \$7, and 160 beans for \$19. Manny needs at least 770 beans. What is the minimum amount of money he needs to spend?

**Answer:** $\boxed{95}$

**Solution:**  Note that if Manny buys 160 beans in groups of 50 and 20, he will need to buy 8 groups of 20, which costs \$24, or 2 groups of 50 and 3 groups of 20, which costs \$23. Therefore, Manny should buy as many groups of 160 as possible. Manny can buy at most 4 groups of 160, giving him 640 beans. He needs 130 more beans; if he buys 0 or 2 groups of 50, the remaining number will not be a multiple of 20, so Manny must be 1 group of 50 and 4 groups of 20. This gives him a final price of \$95.

**8**  A positive integer is "inspirational" if it has at least three factors and the sum of its three smallest positive factors is 12. How many inspirational numbers are less than 2024?

**Answer:** $\boxed{0}$

**Solution:** The smallest factor of any number is 1, so the two next smallest factors must sum to 11. We can check each pair of positive integers that add to 11.

- 2 and 9 don't work because if 9 is a factor, 3 must be a factor.
- 3 and 8 don't work because if 8 is a factor, 2 must be a factor
- 4 and 7 don't work because if 4 is a factor, 2 must be a factor
- 5 and 6 don't work because if 6 is a factor, 2 must be a factor

Therefore, it's impossible to have the 3 smallest factors of a number sum to 12.