

Solutions to Brahmagupta Geometry

- 1 Bob has a rectangular pie. His pie has a width of 4, and a length of 5. What is the area of his pie?

Answer: $\boxed{20}$

Solution: The area of a rectangle with length l and width w is $l \cdot w$. Then, the area of Bob's pie is $4 \cdot 5 = 20$.

- 2 Betty travels 6 parsecs north and 8 parsecs west to get to school. August leaves from the same location as Betty except he travels in a straight line directly to school. How much more distance did Betty travel than August?

Answer: $\boxed{4}$

Solution: Betty travels along the legs of a right triangle with legs of length 6 and 8. Betty travels a total distance of $6 + 8 = 14$. From the Pythagorean theorem, August travels a distance of $\sqrt{6^2 + 8^2} = 10$. Thus, Betty travels $14 - 10 = 4$ more parsecs than August.

- 3 Linda has a plot of land with an area of 16 square meters. If she wants to create four congruent square pens for her farm animals using all her land, how much fencing will she need in meters?

Answer: $\boxed{24}$

Solution: The side length of the square is $\sqrt{16} = 4$ meters. Dividing into four congruent squares and counting, we see that we need 12 fences, each of which has length 2. Thus, the total amount of fencing needed is $2 \cdot 12 = 24$.

- 4 5 congruent squares with side length 2 are packaged together below. The middle square is offset by 45 degrees and is tangent to each of the other squares at the midpoint of each of its sides. Find the distance between the marked corners.

Answer: $\boxed{3}$

Solution: Draw a diagonal of the large square; this diagonal runs through two diagonals and a side length of the smaller square. As such, the length of the large diagonal is $2\sqrt{2} \cdot 2 + 2 = 2 + 4\sqrt{2}$. The side length is therefore $4 + \sqrt{2}$. The two legs have lengths $2 + \frac{\sqrt{2}}{2}$ and $2 - \frac{\sqrt{2}}{2}$, so using the Pythagorean Theorem the length between the two vertices has length

$$\sqrt{\left(2 + \frac{\sqrt{2}}{2}\right)^2 + \left(2 - \frac{\sqrt{2}}{2}\right)^2} = 3.$$

- 5 In the diagram below, lightly shaded donuts with inner radius 1 and outer radius 2 are connected by a darkly shaded rectangular strip with width 1. Given that the area in the darkly shaded section is the same as the area of the lightly shaded sections put together, find the height of the darkly shaded region.

Answer: $\boxed{3\pi}$

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Solution: The area of the lightly shaded sections together is just the area of a circle with radius 2 with a hole of radius 1 inside. This shape has area of $4\pi - \pi = 3\pi$. The rectangle has width 1 and area 3π , so the height of the rectangle must be 3π .

- 6 In triangle XYZ, two sides are 5 and 10 units long, and the angle between them is 60 degrees. Find the area of the triangle.

Answer: $\boxed{\frac{25\sqrt{3}}{2}}$

Solution: The area of a triangle given two sides and the angle *between* the two sides is $\frac{1}{2}ab\sin\theta$, where a and b are the two side lengths and θ is the angle between. Therefore, the area of XYZ is $A = \frac{1}{2} \cdot 5 \cdot 10 \cdot \sin 60^\circ$. We substitute $\sin 60^\circ = \frac{\sqrt{3}}{2}$ into A and get $A = \frac{1}{2} \cdot 5 \cdot 10 \cdot \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{2}$ square units.

- 7 In triangle ABC, points D and E lie on sides AB and AC, respectively, such that $AD = DE = BE = BC$. If $\angle BAC = \angle CBE$, what is $\angle BED$ in degrees?

Answer: $\boxed{\frac{540}{7}}$

Solution: Let $\angle ABC$ be equal to x . Since BEC is isosceles, $\angle CBE = 180 - 2x$. Since $\angle BAC = \angle CBE$, we can find that $\angle ABC = x$. Thus, $\angle EBD = \angle EDB = 3x - 180$. Lastly, we know that $2\angle DAE = \angle EDB$, giving $360 - 4x = 3x - 180$. As such, $x = \frac{540}{7}$. Then $\angle BED$ can be found to be $540 - 6x = \frac{540}{7}$.

- 8 Arnold the Ant starts at the top of a regular octahedron with side length 2. What is the shortest distance Arnold needs to walk to reach the opposite corner, given that he can only travel along the surface of the octahedron?

Answer: $\boxed{2\sqrt{3}}$

Solution: Arnold must first reach the perimeter of the horizontal square boundary, then proceed from there to reach the bottom vertex. The shortest possible distance to the horizontal square boundary is $\sqrt{3}$, the altitude of the face equilateral triangle. The shortest path from the horizontal square boundary to the bottom is also $\sqrt{3}$ and starts at the same point that the top path ends. As such, the shortest total possible distance is simply $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$.