1 Dami splits 2024 in its four digits and randomly chooses one. What is the chance that she picks 4?

Answer: $\boxed{\dfrac{1}{4}}$

Solution: There are 4 digits in 2024, of which only one of them is a 4. Thus, the probability of picking a 4 is $\frac{1}{4}$.

2 Jenny bought 30 of mystery boxes and received 6 large prizes. If the probability of winning a large prize stays the same, how many large prizes is she expected to win from 20 mystery boxes?

Answer: $\boxed{4}$

Solution: The probability of receiving a large prize is $\frac{6}{30} = \frac{1}{5}$. Thus, the expected number of large prizes from opening 20 boxes is $\frac{1}{5} \cdot 20 = 4$.

3 Carlos is picking out his lumberjack outfit. For a given outfit, he needs one pair of pants and one flannel. If Carlos has four pairs of pants and four flannels, each in red, green, blue, and purple, how many different outfits can he have if he does not want to ever wear the same color pants and flannels?

Answer: $\boxed{12}$

Solution: Carlos can pick four colors of pants. If he picks any one color (say red), he can only pick three flannels (not red). Then the total number of outfits he can have is $4 \cdot 3 = 12$.

4 Every day, Mr. Schwartz has a 30% chance of chucking a ball of paper at a sleeping student. What is the probability that he doesn't throw paper balls for 2 consecutive days? Express your answer as a common fraction.

Answer: $\boxed{\dfrac{49}{100}}$

Solution: The probability that he does NOT through a ball on each day is $1 - \frac{3}{10} = \frac{7}{10}$. Then the probability Mr. Schwartz does not throw a ball two days in a row is $(\frac{7}{10})^2 = \frac{49}{100}$.

5 I have a fair coin and an unfair coin. If I draw a coin at random and the probability I flip heads is $\frac{5}{12}$, what is the probability I get heads on my unfair coin?

Answer: $\boxed{\dfrac{1}{3}}$

Solution: Let the probability of flipping heads on an unfair coin be p. Then, from the information given,

$$\frac{1}{2} \cdot p + \frac{1}{2} \cdot \frac{1}{2} = 5/12 \Longrightarrow p = \frac{1}{3}.$$

6 Jason has a fair coin. Evan has a weighted coin where the probability of landing heads is $\frac{3}{4}$. What is the minimum number of flips such that the probability that Evan lands tails for exactly one of his flips is more than twice the probability that Jason lands tails for exactly one of his flips?

Answer: $\boxed{5}$

Solution: Let's say we flip n times. Then the probability Jason lands tails exactly once is $n \cdot (\frac{1}{2})^n$, and the probability Evan lands tails exactly once is $n \cdot (\frac{3}{4})^{n-1} \cdot \frac{1}{4}$. The inequality then becomes $n \cdot (\frac{3}{4})^{n-1} \cdot \frac{1}{4} > 2 \cdot n \cdot (\frac{1}{2})^n$. Simplifying gives

$$(\frac{3}{4})^{n-1} > 4 \cdot (\frac{1}{2})^{n-1} \implies (\frac{3}{2})^{n-1} > 4$$

Testing values gives $n = 5$.

7 Yun and Yi are playing cards. They take turns drawing cards from a standard 52-card deck with replacement, and the first person to draw one of the thirteen clubs wins. What is the probability that Yi will win?

Answer: $\boxed{\frac{3}{7}}$

Solution: Let the probability of a person winning on their turn be p. Then, when the game begins, the probability Yun wins is just p. If Yun wins on the first draw the game is over. If Yun does not win on the first draw the game goes to Yi, and the probability Yi wins is now p. Thus, the probability Yi wins at the start is $\frac{3}{4}p$. The total probability must sum to 1 so $p + \frac{3}{4}p = \frac{7}{4}p = 1$. Solving gives $p = \frac{4}{7}$, so the probability Yun wins is $1 - \frac{4}{7} = \frac{3}{7}$.

8 Lewis is learning to spell. He writes out all of the arrangements of the letters in his name and orders them alphabetically. For example, the first arrangement would be "eilsw". What position is the word "lewis" in?

Answer: $\boxed{53}$

Solution: The ordering of the letters alphabetically is "e", "i", "l", "s", and "w". All words that start with "e" and "i" are before "lewis"; there are a total of $4! \cdot 2 = 48$ such words. Upon looking at words starting with "l", all words starting with "lei" and "les" are before "lewis", for a total of $2! \cdot 2 = 4$ such words. We're now at "lew", and there are no other words before "lewis" starting with "lew". As such, there are a total of $48 + 4 = 52$ words before "lewis", so "lewis" is the 53rd word.