

MBMT Team Round – Germain

April 16, 2023

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

**DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.**

You are highly encouraged to work with your teammates on the problems in order to solve them.

- 1 **[4]** There are 5 red balls and 3 blue balls in a bag. Alice randomly picks a ball out of the bag and then puts it back in the bag. Bob then randomly picks a ball out of the bag. What is the probability that Alice gets a red ball and Bob gets a blue ball, assuming each ball is equally likely to be chosen?

- 2 **[4]** A circle has radius 6. A smaller circle with the same center has radius 5. What is the probability that a dart randomly placed inside the outer circle is outside the inner circle?

- 3 **[4]** Alex and Jeff are playing against Max and Alan in a game of tractor with 2 standard decks of 52 cards. They take turns taking (and keeping) cards from the combined decks. At the end of the game, the 5s are worth 5 points, the 10s are worth 10 points, and the kings are worth 10 points. Given that a team needs 50 percent more points than the other to win, what is the minimal score Alan and Max need to win?

- 4 **[5]** Bob has a sandwich in the shape of a rectangular prism. It has side lengths 10, 5, and 5. He cuts the sandwich along the two diagonals of a face, resulting in four pieces. What is the volume of the largest piece?

- 5 **[5]** Aven makes a rectangular fence of area 96 with side lengths x and y . John makes a larger rectangular fence of area 186 with side lengths $x + 3$ and $y + 3$. What is the value of $x + y$?

- 6 **[5]** A number is prime if it is only divisible by itself and 1. What is the largest prime number n smaller than 1000 such that $n + 2$ and $n - 2$ are also prime? Note: 1 is not prime.

- 7 **[6]** Sally has 3 red socks, 1 green sock, 2 blue socks, and 4 purple socks. What is the probability she will choose a pair of matching socks when only choosing 2 socks without replacement?

- 8 **[6]** A triangle with vertices at $(0, 0)$, $(3, 0)$, $(0, 6)$ is filled with as many 1×1 lattice squares as possible. How much of the triangle's area is not filled in by the squares?
- 9 **[7]** Let A and B be digits. If $125A^2 + B161^2 = 11566946$ What is $A + B$?
- 10 **[8]** A series of concentric circles w_1, w_2, w_3, \dots satisfy that the radius of $w_1 = 1$ and the radius of $w_n = \frac{3}{4}$ times the radius of w_{n-1} . The regions enclosed in w_{2n-1} but not in w_{2n} are shaded for all integers $n > 0$. What is the total area of the shaded regions?
- 11 **[8]** How many ordered pairs of integers (x, y) satisfy $y^2 - xy + x = 0$?
- 12 **[9]** 10 cards labeled 1 through 10 lie on a table. Kevin randomly takes 3 cards and Patrick randomly takes 2 of the remaining 7 cards. What is the probability that Kevin's largest card is smaller than Patrick's largest card, and that Kevin's second-largest card is smaller than Patrick's smallest card?
- 13 **[9]** N consecutive integers add to 27. How many possible values are there for N ?
- 14 **[10]** A circle with center O and radius 7 is tangent to a pair of parallel lines l_1 and l_2 . Let a third line tangent to circle O intersect l_1 and l_2 at points A and B . If $AB = 18$, find $OA + OB$.

- 15 **[10]** Let

$$M = \prod_{i=0}^{42} (i^2 - 5).$$

Given that 43 doesn't divide M , what is the remainder when M is divided by 43?