

# MBMT Number Theory Round – Germain

April 16, 2023

Full Name \_\_\_\_\_

Student ID Number \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- \_\_\_\_\_ 1 Find the largest integer less than 2023 whose square ends in 9.
- \_\_\_\_\_ 2 How many positive integers divide both 100 and 160?
- \_\_\_\_\_ 3 There exist positive integers  $a, b, c$ , with  $b > 1$ , and  $6 \cdot a = b \cdot c = 12000$ . If  $a$  and  $b$  are relatively prime, what is  $c$ ?
- \_\_\_\_\_ 4 What is the largest integer  $n$  such that  $3^n$  is a factor of  $18! + 19! + 20!$ ?
- \_\_\_\_\_ 5 For some positive integer  $1 \leq n \leq 1000$ , Jeremy writes down  $n^2, n^1$ , and  $n^0$  in a row on his whiteboard, in that order. His friend Joshua, however, read the three integers as a single integer and deduced that it is a multiple of 3. For how many  $n$  would this happen?
- \_\_\_\_\_ 6 Suppose we have positive integers that sum up to 200. What is the largest possible product of the integers?
- \_\_\_\_\_ 7 Find the remainder when the sum of  $x(x + 1)(x + 2)$  for all  $x$  ranging from  $x = 1$  to  $x = 39$  is divided by 40.
- \_\_\_\_\_ 8 Find  $x$ , where  $x$  is the remainder when

$$\prod_{k=1}^{40} k!^2$$

is divided by 41.