1 Find $20^3 + 2^2 + 3^1$.

Proposed by Bradley Guo.

Answer: $\boxed{8007}$

Solution: Adding, we find $20^3 + 2^2 + 3 = 8000 + 4 + 3 = 8007$.

2 What is the smallest perfect square that is also a perfect cube?

Proposed by Bradley Guo.

Answer: $\boxed{0}$

Solution: $0^2 = 0^3$. Note no squares are less than 0, so there cannot exist a smaller perfect square.

3 Hanfei spent 14 dollars on chicken nuggets at McDonalds. 4 nuggets cost 3 dollars, 6 nuggets cost 4 dollars, and 12 nuggets cost 9 dollars. How many chicken nuggets did Hanfei buy?

Proposed by Kevin Yao.

Answer: $\boxed{20}$

Solution: If Hanfei buys a set of 9 dollar nuggets, he can only buy at max one other set of nuggets for a maximum of 18 nuggets. However, if he buys two 4 dollar sets and two 3 dollar sets, he can achieve 20 chicken nuggets.

4 What is the radius of a circle with area 4?

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{2}{\sqrt{\pi}}}$

Solution: The area of a circle is πr^2, so $\pi r^2 = 4$, or $r^2 = \frac{4}{\pi}$. Solving, we get $r = \frac{2}{\sqrt{\pi}}$.

5 Bob likes to make pizzas. Bab also likes to make pizzas. Bob can make a pizza in 20 minutes. Bab can make a pizza in 30 minutes. If Bob and Bab want to make 50 pizzas in total, how many hours would that take them?

Proposed by Megan Gu.

Answer: $\boxed{10}$

Solution: If Bob can bake a pizza in 20 minutes, then he can bake 3 pizzas every hour. Similarly, Bab can bake 2 pizzas every hour. Together, they can bake 5 pizzas every hour, so it will take $\frac{50}{5} = 10$ hours.

6 Find the area of an equilateral rectangle with perimeter 20.

Proposed by Bradley Guo.

Answer: $\boxed{25}$

Solution: Because the rectangle is equilateral, each side has length $\frac{20}{4} = 5$, so the area of the rectangle will be $5 \cdot 5 = 25$.

7 What is the minimum possible number of divisors that the sum of two prime numbers greater than 2 can have?

Proposed by Bradley Guo.

Answer: $\boxed{4}$

Solution: The sum of two primes greater than 2 is even, so 1, 2, $\frac{p+q}{2}$, and $p+q$ must be divisors of $p+q$ given that $\frac{p+q}{2}$ isn't equal to 2. Since $3+5=8$ has 4 divisors, this is possible and is the minimum.

8 Kwu and Kz play rock-paper-scissors-dynamite, a variant of the classic rock-paper-scissors in which dynamite beats rock and paper but loses to scissors. The standard rock-paper-scissors rules apply, where rock beats scissors, paper beats rock, and scissors beats paper. If they throw out the same option, they keep playing until one of them wins. If Kz randomly throws out one of the four options with equal probability, while Kwu only throws out dynamite, what is the probability Kwu wins?

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{2}{3}}$

Solution: Kz has to eventually throw out rock, paper, or scissors. Dynamite beats two out of those three options.

9 Aven has 4 distinct baguettes in a bag. He picks three of the bagged baguettes at random and lays them on a table in random order. How many possible orderings of three baguettes are there on the table?

Proposed by Jason Youm.

Answer: $\boxed{24}$

Solution: First, we find the number of ways Aven can choose three baguettes from his bag. He can do so in $\binom{4}{3} = 4$ ways. Next, we find the number of orderings of three baguettes. Aven can order them in $3! = 6$ ways. Since these events are independent, we multiply to get our final answer of $4 \times 6 = 24$ orderings.

10 Find the largest 7-digit palindrome that is divisible by 11.

Proposed by Evan Wu.

Answer: $\boxed{9997999}$

Solution: 9900099 is clearly divisible by 11. Then, we just need the largest 3-digit palindrome divisible by 11, which is 979.

11 Let triangle ABC be an equilateral triangle with side length 6. If point D is on $\overline{AB}$ and point E is on $\overline{BC}$, find the minimum possible value of $AD + DE + CE$.

Proposed by Bradley Guo.

Answer: $\boxed{6}$

Solution: From the triangle inequality, we know that $AD + DE + CE \leq AD + CD \leq AC$. This is satisfied when $D = A$ and $E = C$. The answer is $AC = 6$.

12 Find the smallest positive integer n with at least seven divisors.

Proposed by Evan Wu.

Answer: $\boxed{24}$

Solution: $24 = 2^3 \cdot 3$, giving $4 \cdot 2 = 8$ divisors. All smaller numbers have fewer divisors.

13 Square A is inscribed in a circle. The circle is inscribed in Square B. If the circle has a radius of 10, what is the ratio between a side length of Square A and a side length of Square B?

Proposed by Autumn Qiu.

Answer: $\boxed{\dfrac{\sqrt{2}}{2}}$

Solution: If a square is inscribed in a circle, the square's diagonal will equal the circle's diameter. So Square A's diagonal is 20. If its diagonal is 20, its side length is $10 \cdot \sqrt{2}$ by Pythagorean theorem. When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square. Thus, Square B's side length is 20. $\frac{10\sqrt{2}}{20}$ is simplified to be $\frac{\sqrt{2}}{2}$.

14 Billy Bob has 5 distinguishable books that he wants to place on a shelf. How many ways can he order them if he does not want his two math books to be next to each other?

Proposed by Evan Zhang.

Answer: $\boxed{72}$

Solution: There are a total of $5! = 120$ ways for him to place the 5 books on the shelf without restrictions. We can count how many ways there are for him to place the books such that the two math books are next to each other. There are $2 * 4! = 48$ ways to place the books in this manner, as we can treat the two math books as one unit. Therefore, there are 72 ways to place the books with these restrictions.

15 Six people make statements as follows: Person 1 says "At least one of us is lying."

Person 2 says "At least two of us are lying."

Person 3 says "At least three of us are lying."

Person 4 says "At least four of us are lying."

Person 5 says "At least five of us are lying."

Person 6 says "At least six of us are lying."

How many are lying?

Proposed by Bradley Guo.

Answer: $\boxed{3}$

Solution: Assume n of them are lying. Then, people numbered between 1 and n inclusive will be telling the truth, meaning that 6-n will be lying. Thus, 6-n = n, so n = 3.

16 If x and y are between 0 and 1, find the ordered pair (x, y) which maximizes $(xy)^2(x^2-y^2)$

Proposed by Heerok Das.

Answer: $\boxed{(1, \frac{\sqrt{2}}{2})}$

Solution: Let $a = x^2, b = y^2$. We want to maximize $x^2y^2(x^2 - y^2) = ab(a - b)$. Clearly this is increasing in a, so we set $a = 1$ to get $b(1 - b)$. We can complete the square to get $\frac{1-(2b-1)^2}{4}$, which is maximized when $b = \frac{1}{2}$, which corresponds to $y = \frac{\sqrt{2}}{2}$.

17 If I take all my money and divide it into 12 piles, I have 10 dollars left. If I take all my money and divide it into 13 piles, I have 11 dollars left. If I take all my money and divide it into 14 piles, I have 12 dollars left. What's the least amount of money I could have?

Proposed by Daniel He.

Answer: $\boxed{1090}$

Solution: If I had 2 more dollars, it would be divisible by 12, 13, and 14. The smallest number with this property is $12 \cdot 13 \cdot 7 = 1092$, so I have 1090 dollars.

18 A quadratic equation has two distinct prime number solutions and its coefficients are integers that sum to a prime number. Find the sum of the solutions to this equation.

Proposed by Vijay Shanmugam.

Answer: $\boxed{5}$

Solution: Let $f(x) = ax^2 + bx + c = 0$ have solutions p, q. Note $p + q = \frac{-b}{a}$. $a = 1$ because p, q are prime, so $p + q = -b$. We also know $p * q = \frac{c}{a}$, so $p * q = c$. We also know that $a + b + c =$ prime number, so $1 + b + c =$ prime number. Substituting, we get $1 - (p + q) + (p * q) =$ prime number, so $(p - 1)(q - 1) =$ prime number, which means $p = 2$ and $q - 1 =$ prime number. Since q is also prime, $q = 3$, so $p + q = 5$.

19 Let $ABCD$ be a square of side length 2. Let M be the midpoint of AB and N be the midpoint of AD. Let the intersection of BN and CM be E. Find the area of quadrilateral $NECD$.

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{11}{5}}$

Solution: Let the area of triangle $\triangle MEB$ be x. We know that $\angle MEB = 90$ because NB has slope $\frac{1}{2}$ and MC has slope -2. $\triangle MEB$ and $\triangle BEC$ are similar with a factor of 2, so the area of $\triangle BEC = 4x$. Quadrilateral AMEN also has area $4x$ because triangles ABN and MBC have the same area. The area of square ABCD is four times the area of triangle MBC, which has area $5x$. So, $x = \frac{1}{5}$. Summing the individual pieces up, we find quadrilateral NECD has area $\frac{11}{5}$.

20 A regular 12-sided polygon is inscribed in a circle. Gaz then chooses 3 vertices of the polygon at random and connects them to form a triangle. What is the probability that this triangle is right?

Proposed by Kian Dhawan.

Answer: $\boxed{\dfrac{3}{11}}$

Solution: First note that, by the inscribed angle theorem, for the triangle to be right, one of the sides has to be the diameter of the circle. There are 6 ways to choose which diameter the side is, and for each of these, the third point can be any of the remaining 10 vertices. This gives 60 possible right triangles. There are a total of $\binom{12}{3} = 220$ to pick the three vertices, so there are 220 possible total triangles. The probability the triangle is right $\frac{60}{220} = \frac{3}{11}$.

21 Quadrilateral ABCD has $\angle A = \angle D = 60°$. If $AB = 8$, $CD = 10$, and $BC = 3$, what is length AD?

Proposed by Tony Song.

Answer: $\boxed{9 + \sqrt{6}}$

Solution: Let E and F be the points resulting from dropping B and C to AD. Note ABE and CDF are $30-60-90$ right triangles. Because $AB = 8$ and $CD = 10$, we can find that $AE = 4$ and $DF = 5$. To evaluate EF, we can drop B to some point X on FC. $BCFE$ is a trapezoid, so $CX = CF - BE = 5\sqrt{3} - 4\sqrt{3} = \sqrt{3}$, so $EF = \sqrt{3^2 - \sqrt{3}^2} = \sqrt{6}$. Thus, $AD = 4 + \sqrt{6} + 5 = 9 + \sqrt{6}$.

22 A book has at most 7 chapters, and each chapter is either 3 pages long or has a length that is a power of 2 (including 1). What is the least positive integer n for which the book cannot have n pages?

Proposed by Kelin Zhu.

Answer: $\boxed{509}$

Solution: n's binary representation must contain at least eight 1's, since otherwise, it is expressible as a sum of at most seven distinct powers of two. However, since $3_{10} = 11_2$, the binary expansion of n also must have at least seven 1's before the final two digits. The least possible such n is $111111101_2 = 509$.

23 $\triangle ABC$ is an equilateral triangle of side length x. Three unit circles ω_A, ω_B, and ω_C lie in the plane such that ω_A passes through A while ω_B and ω_C are centered at B and C, respectively. Given that ω_A is externally tangent to both ω_B and ω_C, and the center of ω_A is between point A and line $\overline{BC}$, find x.

Proposed by Jeffrey Tong.

Answer: $\boxed{\dfrac{\sqrt{3}+\sqrt{15}}{2}}$

Solution: Let the center of ω_A be O and the midpoint of BC be D. We know that $BD = \frac{x}{2}$, $OD = AD - AO = \frac{x\sqrt{3}}{2} - 1$, and $BO = 2$. Pythagorean theorem on triangle BDO gives the equation $x^2 - \sqrt{3}x - 3 = 0$. The quadratic formula gives the answer $x = \frac{\sqrt{3}+\sqrt{15}}{2}$.

24 For some integers n, the quadratic function $f(x) = x^2 - (6n-6)x - (n^2 - 12n + 12)$ has two distinct positive integer roots, exactly one out of which is a prime and at least one of which is in the form 2^k for some nonnegative integer k. What is the sum of all possible values of n?

Proposed by Kelin Zhu.

Answer: $\boxed{7}$

Solution: There are two cases: the roots are $1, p$ or $2, 2m$ for prime p and positive integer m. In case one, we have $p = 6n-7$ by Vieta's, and also $6n-7 = -(n^2-12n+12)$; solving the equation gives $n = 1, 5$, however, $n = 1$ gives $p = -1$ which cannot happen. In case two, we have $2m = 6n - 8$ and $12n - 16 = -(n^2 - 12n + 12)$; solving this gives $m = -2, 2$, the former of which again violates the positive roots condition. The answer is therefore $5 + 2 = 7$.

25 In a triangle, let the altitudes concur at H. Given that $AH = 30, BH = 14$, and the circumradius is 25, calculate CH .

Proposed by Kevin Wu.

Answer: $\boxed{30}$

Solution: Let the reflections of the orthocenter over BC, AC, AB be H_A, H_B, H_C respectively, it is well known that these three points are all on the circumcircle of (ABC). Thus, $AH_BCH_ABH_C$ is a hexagon with circumradius 25, and side lengths $30, CH, CH, 14, 14, 30$. We can reorder the sides to get that there is another hexagon $P_1P_2P_3P_4P_5P_6$ with $P_1P_2 = 30, P_2P_3 = CH, P_3P_4 = 14, P_4P_5 = 14, P_5P_6 = CH, P_6P_1 = 30$, and circumradius 25. However, by symmetry we now have that P_1, P_4 are diametrically opposite, so then $P_1P_4 = 50$. Now, by Pythagorean theorem $P_2P_4 = 40, P_1P_3 = 48$. Finally, Ptolemy's gives that $CH = P_2P_3 = \frac{48\cdot 40 - 14\cdot 30}{50} = 30$.

26 What percent of the problems on the individual, team, and guts rounds for both divisions have integer answers?

Proposed by Bradley Guo.

Answer: $\boxed{65.65656565656566}$

Solution: N/A

27 Estimate $12345^{\frac{1}{123}}$.

Proposed by Kevin Wu.

Answer: $\boxed{1.07960318}$

Solution: N/A

28 Let O be the center of a circle ω with radius 3. Let A, B, C be randomly selected on ω. Let M, N be the midpoints of sides BC, CA, and let AM, BN intersect at G. What is the probability that $OG \leq 1$?

Proposed by Bradley Guo and Kevin Wu.

Answer: $\boxed{.25}$

Solution: N/A

29 Let $r(a, b)$ be the remainder when a is divided by b. What is $\sum_{i=1}^{100} r(2^i, i)$?

Proposed by Kevin Wu.

Answer: $\boxed{1576}$

Solution: N/A

30 Bongo flips 2023 coins. Call a run of heads a sequence of consecutive heads. Say a run is maximal if it isn't contained in another run of heads. For example, if he gets $HHHTTHTTHHHHTH$, he'd have maximal runs of length $3, 1, 4, 1$. Bongo squares the lengths of all his maximal runs and adds them to get a number M. What is the expected value of M?

Proposed by Kevin Wu.

Answer: $\boxed{3032.5 + 2^{-2022}}$

Solution: N/A