MBMT Germain Guts Round — Set 1
April 16, 2023

1 **[3]** Find $20^3 + 2^2 + 3^1$.

2 **[3]** What is the smallest perfect square that is also a perfect cube?

3 **[3]** Hanfei spent 14 dollars on chicken nuggets at McDonalds. 4 nuggets cost 3 dollars, 6 nuggets cost 4 dollars, and 12 nuggets cost 9 dollars. How many chicken nuggets did Hanfei buy?

4 **[3]** What is the radius of a circle with area 4?

5 **[3]** Bob likes to make pizzas. Bab also likes to make pizzas. Bob can make a pizza in 20 minutes. Bab can make a pizza in 30 minutes. If Bob and Bab want to make 50 pizzas in total, how many hours would that take them?

MBMT Germain Guts Round — Set 2
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6 **[4]** Find the area of an equilateral rectangle with perimeter 20.

7 **[4]** What is the minimum possible number of divisors that the sum of two prime numbers greater than 2 can have?

8 **[4]** Kwu and Kz play rock-paper-scissors-dynamite, a variant of the classic rock-paper-scissors in which dynamite beats rock and paper but loses to scissors. The standard rock-paper-scissors rules apply, where rock beats scissors, paper beats rock, and scissors beats paper. If they throw out the same option, they keep playing until one of them wins. If Kz randomly throws out one of the four options with equal probability, while Kwu only throws out dynamite, what is the probability Kwu wins?

9 **[4]** Aven has 4 distinct baguettes in a bag. He picks three of the bagged baguettes at random and lays them on a table in random order. How many possible orderings of three baguettes are there on the table?

10 **[4]** Find the largest 7-digit palindrome that is divisible by 11.

MBMT Germain Guts Round — Set 3
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11 **[5]** Let triangle ABC be an equilateral triangle with side length 6. If point D is on $\overline{AB}$ and point E is on $\overline{BC}$, find the minimum possible value of $AD + DE + CE$.

12 **[5]** Find the smallest positive integer n with at least seven divisors.

13 **[5]** Square A is inscribed in a circle. The circle is inscribed in Square B. If the circle has a radius of 10, what is the ratio between a side length of Square A and a side length of Square B?

14 **[5]** Billy Bob has 5 distinguishable books that he wants to place on a shelf. How many ways can he order them if he does not want his two math books to be next to each other?

15 **[5]** Six people make statements as follows: Person 1 says "At least one of us is lying."

Person 2 says "At least two of us are lying."

Person 3 says "At least three of us are lying."

Person 4 says "At least four of us are lying."

Person 5 says "At least five of us are lying."

Person 6 says "At least six of us are lying."

How many are lying?

16 **[7]** If x and y are between 0 and 1, find the ordered pair (x, y) which maximizes $(xy)^2(x^2 - y^2)$

17 **[7]** If I take all my money and divide it into 12 piles, I have 10 dollars left. If I take all my money and divide it into 13 piles, I have 11 dollars left. If I take all my money and divide it into 14 piles, I have 12 dollars left. What's the least amount of money I could have?

18 **[7]** A quadratic equation has two distinct prime number solutions and its coefficients are integers that sum to a prime number. Find the sum of the solutions to this equation.

19 **[7]** Let $ABCD$ be a square of side length 2. Let M be the midpoint of AB and N be the midpoint of AD. Let the intersection of BN and CM be E. Find the area of quadrilateral $NECD$.

20 **[7]** A regular 12-sided polygon is inscribed in a circle. Gaz then chooses 3 vertices of the polygon at random and connects them to form a triangle. What is the probability that this triangle is right?

21 **[9]** Quadrilateral ABCD has $\angle A = \angle D = 60°$. If $AB = 8$, $CD = 10$, and $BC = 3$, what is length AD?

22 **[9]** A book has at most 7 chapters, and each chapter is either 3 pages long or has a length that is a power of 2 (including 1). What is the least positive integer n for which the book cannot have n pages?

23 **[9]** $\triangle ABC$ is an equilateral triangle of side length x. Three unit circles ω_A, ω_B, and ω_C lie in the plane such that ω_A passes through A while ω_B and ω_C are centered at B and C, respectively. Given that ω_A is externally tangent to both ω_B and ω_C, and the center of ω_A is between point A and line $\overline{BC}$, find x.

24 **[9]** For some integers n, the quadratic function $f(x) = x^2 - (6n - 6)x - (n^2 - 12n + 12)$ has two distinct positive integer roots, exactly one out of which is a prime and at least one of which is in the form 2^k for some nonnegative integer k. What is the sum of all possible values of n?

25 **[9]** In a triangle, let the altitudes concur at H. Given that $AH = 30, BH = 14$, and the circumradius is 25, calculate CH .

26 **[12]** What percent of the problems on the individual, team, and guts rounds for both divisions have integer answers?

27 **[12]** Estimate $12345^{\frac{1}{123}}$.

28 **[12]** Let O be the center of a circle ω with radius 3. Let A, B, C be randomly selected on ω. Let M, N be the midpoints of sides BC, CA, and let AM, BN intersect at G. What is the probability that $OG \leq 1$?

29 **[12]** Let $r(a, b)$ be the remainder when a is divided by b. What is $\sum_{i=1}^{100} r(2^i, i)$?

30 **[12]** Bongo flips 2023 coins. Call a run of heads a sequence of consecutive heads. Say a run is maximal if it isn't contained in another run of heads. For example, if he gets $HHHTTHTTHHHHTH$, he'd have maximal runs of length $3, 1, 4, 1$. Bongo squares the lengths of all his maximal runs and adds them to get a number M. What is the expected value of M?