1 There are two boxes: one red and one white. How many ways are there to put 30 identical blue balls in the two boxes?

Proposed by Michelle Gao.

Answer: $\boxed{31}$

Solution: Somewhere from 0 to 30 blue balls go in the red box, and the rest go in the white box. Therefore, there are 31 possibilities.

2 You are invited to play the "octopus game". One of the games requires you to jump over 8 rows of tiles. Each row has 2 tiles, one of which is safe (the other will land you harmlessly into the octopus tank). Calculate the probability that you will make it through if you are the first to go.

Proposed by Skylar Xue.

Answer: $\boxed{\dfrac{1}{256}}$

Solution: There's a $\frac{1}{2}$ probability we will survive any given jump. Since there are 8 of them, we get $(\frac{1}{2})^8 = \frac{1}{256}$.

3 A sock drawer has 4 red, 4 blue, and 4 yellow socks. How many socks does Patrick have to take out to ensure he has at least two pairs?

Proposed by Kevin Yao.

Answer: $\boxed{6}$

Solution: Given 3 socks, the only way to have no pairs is to have one of each color. When we take the 4th sock, we are guaranteed at least one pair. However, when we take the 5th sock, it is possible that the 5th sock is the same color as the previous pair, which means that it is possible to not get a pair on the 5th sock. However, now we have 3 pairless socks of each color again, so our 6th sock is guaranteed to give us our second pair.

4 Alex randomly plays a note on his mini 12-key piano. He then randomly plays a different note. What are the chances that the two keys he played are right next to each other on the piano?

Proposed by Evan Wang.

Answer: $\boxed{\dfrac{1}{6}}$

Solution: There are 12 ways to pick the first key and 11 ways to pick the second. There are also 22 ways to play two keys next to each other: we can pick keys $(1,2),(2,3),\ldots,(11,12)$ or $(2,1),(3,2),\ldots,(12,11)$. Therefore the probability is $\frac{22}{12\cdot 11} = \frac{1}{6}$.

5 Whenever Evan tries to pronounce a word with a T in it, he ignores all the Ts. How many unique pronunciations of a 6-letter word can Evan make, given that the word can only contain the letters M, B, and T? For example, Evan pronounces 'MMBMMT' the same as 'MMTBMM', but he pronounces 'BBTBBB' differently from 'BBTBTB'. Note that 'TTTTTT' is a distinct pronunciation.

Proposed by Bradley Guo and Nathan Shan.

Answer: $\boxed{127}$

Solution: If there are n Ts in the word, after removing them there are $6-n$ letters, each of which can either be a B or an M, so there are 2^{6-n} possibilities. Summing from $n=0$ to $n=6$ gives a total of 127 unique pronunciations.

6 Kwu is trying to flip heads. If he ever flips tails, the god of luck will pity Kwu and guarantee his next flip to be heads. On his first flip, Kwu has a 50 percent chance of flipping heads. What is the probability that his 5th flip lands on heads?

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{21}{32}}$

Solution: Let the probability that Kwu flips heads on his nth flip be P_n. Then on the $n+1$th flip, he gets heads with probability $\frac{1}{2}$ if the nth flip was heads, and probability 1 if he flipped tails, so we have $P_{n+1} = \frac{P_n}{2} + (1 - P_n) = 1 - \frac{P_n}{2}$. Since $P_1 = \frac{1}{2}$, we find that $P_5 = \frac{21}{32}$.

7 Find the number of positive integers less than 1000 that cannot be represented as a sum of two non-negative palindromes, where digits 0 through 9 are considered palindromes. For example, 1 can be written as $1+0$, and 222 can be written as $111+111$, but 21 cannot be written as the sum of two palindromes.

Proposed by Bradley Guo.

Answer: $\boxed{9}$

Solution: If x is a palindrome, then the numbers x through x + 9 can be achieved. For two-digit numbers, x + 11 is also a palindrome. So the only two-digit integers that can't be achieved are numbers of the form x + 10 where x is a two-digit palindrome: $21, 32, \ldots, 98$. For three-digit numbers, x + 10 is a palindrome if the tens digit of x is not 9. This achieves every three-digit integer except 201, 302, 403, 504, 605, 706, 807, and 908. Notice that all can be achieved besides 201 by using 111 and x - 111, while any two or three-digit palindrome when subtracted from 201 gives a multiple of 10. Our solution set is thus $21, 32, \cdots, 98, 201$, of which there are $\boxed{9}$ solutions.

8 On the number line, a man initially stands on the origin. Each second, if he is currently on k where $k < 2023$, he can move to any positive integer n where $k < n \leq 2022$ with a 2^{k-n} chance and can move to 2023 with a 2^{k-2022} chance. When he reaches 2023, he stops moving. What is the probability that he ever is on 2017?

Proposed by Kelin Zhu.

Answer: $\boxed{\dfrac{1}{2}}$

Solution: At any point in time where his integer is less than 2017, he has an equal probability of jumping to 2017 as the sum of probabilities of jumping to $2018, 2019, 2020, 2021, 2022, 2023$. Therefore the probability must be $\frac{1}{2}$.