MBMT Counting and Probability Round — Germain

April 16, 2023

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** There are two boxes: one red and one white. How many ways are there to put 30 identical blue balls in the two boxes?

_____ **2** You are invited to play the "octopus game". One of the games requires you to jump over 8 rows of tiles. Each row has 2 tiles, one of which is safe (the other will land you harmlessly into the octopus tank). Calculate the probability that you will make it through if you are the first to go.

_____ **3** A sock drawer has 4 red, 4 blue, and 4 yellow socks. How many socks does Patrick have to take out to ensure he has at least two pairs?

_____ **4** Alex randomly plays a note on his mini 12-key piano. He then randomly plays a different note. What are the chances that the two keys he played are right next to each other on the piano?

_____ **5** Whenever Evan tries to pronounce a word with a T in it, he ignores all the Ts. How many unique pronunciations of a 6-letter word can Evan make, given that the word can only contain the letters M, B, and T? For example, Evan pronounces 'MMBMMT' the same as 'MMTBMM', but he pronounces 'BBTBBB' differently from 'BBTBTB'. Note that 'TTTTTT' is a distinct pronunciation.

_____ **6** Kwu is trying to flip heads. If he ever flips tails, the god of luck will pity Kwu and guarantee his next flip to be heads. On his first flip, Kwu has a 50 percent chance of flipping heads. What is the probability that his 5th flip lands on heads?

_____ **7** Find the number of positive integers less than 1000 that cannot be represented as a sum of two non-negative palindromes, where digits 0 through 9 are considered palindromes. For example, 1 can be written as $1 + 0$, and 222 can be written as $111 + 111$, but 21 cannot be written as the sum of two palindromes.

_____ **8** On the number line, a man initially stands on the origin. Each second, if he is currently on k where $k < 2023$, he can move to any positive integer n where $k < n \leq 2022$ with a 2^{k-n} chance and can move to 2023 with a 2^{k-2022} chance. When he reaches 2023, he stops moving. What is the probability that he ever is on 2017?