1 Yunyi discovers that when he multiplies a number by 2, he gets the same result as if he added 5 to the number. What is the number?

Proposed by Evan Zhang.

Answer: $\boxed{5}$

Solution: Let Yunyi's number be x. We are given that $2x = x + 5$. Subtracting x from both sides, we get $x = 5$.

2 At noon, a clock shows the correct time. At m minutes after noon, the clock suddenly starts ticking at half the speed it should. At 1:30, the clock shows 1 o'clock. Find the value of m.

Proposed by Ivy Guo.

Answer: $\boxed{30}$

Solution: After 90 minutes, the clock is 30 minutes behind. After the clock slows down, every two minutes, the clock gets one minute behind. It would take 60 minutes for the clock to get 30 minutes behind. Therefore, for the first 30 minutes, the clock must tick at the correct speed, so $m = 30$.

3 Compute $\sqrt{10004 \cdot 9996 + 16}$.

Proposed by Michelle Gao.

Answer: $\boxed{10000}$

Solution: Notice that $10004 = 10000 + 4$ and $9996 = 10000 - 4$. We can then create a difference of squares: $\sqrt{(10000+4)(10000-4)+16} = \sqrt{(10000^2 - 16) + 16} = 10000$.

4 Let a_n be the sum of integers from 1 to n (for instance, $a_1 = 1$, $a_3 = 1 + 2 + 3$). And let $b_n = \frac{a_{2n-1}}{a_{2n}}$. Find $b_1 \cdot b_2 \cdot b_3 \cdot \ldots \cdot b_{10}$.

Proposed by Evan Wang.

Answer: $\boxed{\dfrac{1}{21}}$

Solution: The sum of number 1 to n is given by the formula $\frac{n(n+1)}{2}$. This means $b_n = \frac{\frac{(2n-1)2n}{2}}{\frac{2n(2n+1)}{2}} = \frac{2n-1}{2n+1}$. Thus, the product is equal to $\frac{1}{3} \cdot \frac{3}{5} \cdot \frac{5}{7} \cdot \cdots \cdot \frac{19}{21}$ which simplifies to $\frac{1}{21}$.

5 Suppose Bradley has a sequence such that $x_n = 3x_{n-1} + 2$. If $x_0 = 0$, then what is x_{20}?

Proposed by Kevin Wu.

Answer: $\boxed{3^{20} - 1}$

Solution: Let $y_n + c = x_n$, where c is a constant to be chosen later. Then, we get $y_n + c = 3y_{n-1} + 3c + 2$, or $y_n = 3y_{n-1} + 2c + 2$. Setting $c = -1$, we get

$y_n = 3y_{n-1} + (-2) + 2 = 3y_{n-1}$. Because $x_0 = 0$, we know $y_0 = 1$, and since $y_{20} = y_{19} \cdot 3 = y_{18} \cdot 3^2 = \cdots = y_0 \cdot 3^{20} = 3^{20}$, we can find that $x_{20} = 3^{20} - 1$.

6 Compute $0.25^{0.25^{0.25^{\cdots}}}$ where the number of 0.25's goes to infinity.

Proposed by Bradley Guo.

Answer: $\boxed{0.5}$

Solution: Let x be the quantity in question. We can see that $0.25^x = x$, so we solve to get $x = 0.5$.

7 A function of the form $f(x) = ax^3 + bx^2 + cx + d$ is known to have $f(100) = 2, f(101) = 0, f(102) = 2, f(103) = 3$. Find $f(104)$.

Proposed by Nathan Cho.

Answer: $\boxed{-2}$

Solution: Let us analyze the polynomial $g(x) = f(x + 100) - 2$, which we know has values $0, -2, 0, 1$ at the inputs $0, 1, 2, 3$ respectively. We know it will be in the form $x(x - 2)(px + q)$, since we know two of its roots. Plugging in our inputs to this, we obtain $g(1) = -p - q, g(3) = 9p + 3q$. We want to find $g(4) = 32p + 8q = 4(9p + 3q) + 4(-p - q) = 4 \cdot 1 - 4 \cdot 2 = -4$.

We have $-4 = g(4) = f(104) - 2$, telling us that $f(104) = -2$.

8 Suppose x, y, z are positive reals that satisfy $2x + 3y + 4z = 12$ and $3xyz = 8$. What is $x + y + z$?

Proposed by Kevin Wu.

Answer: $\boxed{\dfrac{13}{3}}$

Solution: Consider the arithmetic mean and geometric mean of $\{2x, 3y, 4z\}$. The arithmetic mean is $\frac{2x+3y+4z}{3} = \frac{12}{3} = 4$. The geometric mean is $\sqrt[3]{2x \cdot 3y \cdot 4z} = \sqrt[3]{24xyz} = 4$.

By AM-GM, we know that for these two to be equal, then $2x = 3y = 4z = 4$, so then $x = 2$, $y = \frac{4}{3}$, $z = 1$, giving $x + y + z = \frac{13}{3}$.