**1** What is the sum of the first 5 positive integers?

*Proposed by Bradley Guo.*

**Answer:** $\boxed{15}$

**Solution:** We can add them all up to get $1+2+3+4+5=15$.

**2** Bread picks a number $n$. He finds out that if he multiplies $n$ by 23 and then subtracts 20, he gets 46279. What is $n$?

*Proposed by Kevin Wu.*

**Answer:** $\boxed{2013}$

**Solution:** We get that $n \cdot 23 - 20 = 46279$, so then $n = \frac{46279+20}{23} = 2013$.

**3** A Harshad Number is a number that is divisible by the sum of its digits. For example, 27 is divisible by $2+7=9$. Only one two-digit multiple of 9 is not a Harshad Number. What is this number?

*Proposed by Isabelle Yang.*

**Answer:** $\boxed{99}$

**Solution:** Go through the two-digit multiples of nine. Every two-digit multiple of 9, except for 99, has digits that sum to nine itself. Since the numbers are multiples of nine, they are also divisible by 9. The exception is 99 since $9+9=18$ and $18 \nmid 99$.

**4** There are 5 red balls and 3 blue balls in a bag. Alice randomly picks a ball out of the bag and then puts it back in the bag. Bob then randomly picks a ball out of the bag. What is the probability that Alice gets a red ball and Bob gets a blue ball, assuming each ball is equally likely to be chosen?

*Proposed by Valerie Song.*

**Answer:** $\boxed{\dfrac{15}{64}}$

**Solution:** We can first calculate the probability of Alice getting a red ball. There are 5 red balls and 8 total, so the probability of Alice getting a red ball is $\frac{5}{8}$. Similarly, since Alice puts her ball back into the bag, the probability that Bob gets a blue ball is $\frac{3}{8}$. These events are independent, so the probability of both events happening is $\frac{5}{8} \cdot \frac{3}{8} = \frac{15}{64}$.

**5** Let $a$ be a 1-digit positive integer and $b$ be a 3-digit positive integer. If the product of $a$ and $b$ is a 4-digit integer, what is the minimum possible value of the sum of $a$ and $b$?

*Proposed by Bradley Guo.*

**Answer:** $\boxed{121}$

**Solution:** If $a=9$, the minimal possible value of $b$ such that $a \cdot b$ is four digits is 112. This gives us a sum of $9+112=121$. Note that for any $a$ less than 9, $b$ will be larger than 125, so $a+b>121$.

**6** A circle has radius 6. A smaller circle with the same center has radius 5. What is the probability that a dart randomly placed inside the outer circle is outside the inner circle?

*Proposed by Tony Song.*

**Answer:** $\boxed{\dfrac{11}{36}}$

**Solution:** The area of the outer circle is $36\pi$, while the area of the inner circle is $25\pi$, so the area inside the outer circle but outside the inner circle is $11\pi$, so the chance of landing in the desired area is $\frac{11\pi}{36\pi} = \frac{11}{36}$.

**7** Call a two-digit integer "sus" if its digits sum to 10. How many two-digit primes are sus?

*Proposed by William Zhang.*

**Answer:** $\boxed{3}$

**Solution:** The only two-digit sus integers are 19, 28, 37, 48, 55, 64, 73, 82, and 91. Out of these, 19, 37, and 73 are prime.

**8** Alex and Jeff are playing against Max and Alan in a game of tractor with 2 standard decks of 52 cards. They take turns taking (and keeping) cards from the combined decks. At the end of the game, the 5s are worth 5 points, the 10s are worth 10 points, and the kings are worth 10 points. Given that a team needs 50 percent more points than the other to win, what is the minimal score Alan and Max need to win?

*Proposed by Kevin Wu.*

**Answer:** $\boxed{120}$

**Solution:** There are $5 + 10 + 10 = 25$ points in a suit, so there are $25 \cdot 4 = 100$ points in a deck and $100 \cdot 2 = 200$ points in the whole game. If Alan and Max's score is $x$, we need $x \geq (200 - x)\frac{3}{2}$, so then $x + \frac{3}{2}x \geq 300$, so we get $x \geq \boxed{120}$.

**9** Bob has a sandwich in the shape of a rectangular prism. It has side lengths 10, 5, and 5. He cuts the sandwich along the two diagonals of a face, resulting in four pieces. What is the volume of the largest piece?

*Proposed by Joshua Hsieh.*

**Answer:** $\boxed{62.5}$

**Solution:** No matter how the sandwich is arranged, cutting along the two diagonals will result in four pieces with the same volume. The total volume of sandwich is $10 \cdot 5 \cdot 5 = 250$, so the volume of any given piece is $\frac{250}{4} = \frac{125}{2}$.

**10** Aven makes a rectangular fence of area 96 with side lengths $x$ and $y$. John makes a larger rectangular fence of area 186 with side lengths $x+3$ and $y+3$. What is the value of $x+y$?

*Proposed by Kian Dhawan.*

**Answer:** $\boxed{27}$

**Solution:** From the areas, we get the two equations $xy = 96$ and $(x+3)(y+3) = 186$. Expanding the second one, we get $xy+3x+3y+9 = 186$. Substituting the first equation into this, $96+3x+3y+9 = 186$. Simplifying, we get $3x+3y = 81$ or $x+y = 27$.

**11** A number is prime if it is only divisible by itself and 1. What is the largest prime number $n$ smaller than 1000 such that $n+2$ and $n-2$ are also prime? Note: 1 is not prime.

*Proposed by Heerok Das.*

**Answer:** $\boxed{5}$

**Solution:** If $n$ is one more than a multiple of $3, n+2$ is divisible by 3. If $n$ is two more than a multiple of $3, n-2$ is divisible by 3. The only prime divisible by 3 is 3 itself, so to maximize $n$, we must have $n-2 = 3$, or $n = 5$.

**12** Sally has 3 red socks, 1 green sock, 2 blue socks, and 4 purple socks. What is the probability she will choose a pair of matching socks when only choosing 2 socks without replacement?

*Proposed by Sophia Sun and Megan Gu.*

**Answer:** $\boxed{\dfrac{2}{9}}$

**Solution:** There are 3 separate cases in which you choose a red pair, a blue pair, or purple pair. Evaluating each and summing them, we have $\frac{3}{10} \cdot \frac{2}{9} + \frac{2}{10} \cdot \frac{1}{9} + \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{9}$

**13** A triangle with vertices at $(0,0)$, $(3,0)$, $(0,6)$ is filled with as many $1 \times 1$ lattice squares as possible. How much of the triangle's area is not filled in by the squares?

*Proposed by Kevin Yao.*

**Answer:** $\boxed{3}$

**Solution:** Only 6 squares can be filled by lattice squares, so $\frac{3 \cdot 6}{2} - 6 = 9 - 6 = 3$ are not filled in.

**14** A series of concentric circles $w_1, w_2, w_3, ...$ satisfy that the radius of $w_1 = 1$ and the radius of $w_n = \frac{3}{4}$ times the radius of $w_{n-1}$. The regions enclosed in $w_{2n-1}$ but not in $w_{2n}$ are shaded for all integers $n > 0$. What is the total area of the shaded regions?

*Proposed by Yunyi Ling.*

**Answer:** $\boxed{\dfrac{16\pi}{25}}$

**Solution:** $A = \pi(1 - (\frac{3}{4})^2 + (\frac{3}{4})^4 - (\frac{3}{4})^6 + (\frac{3}{4})^8 - (\frac{3}{4})^{10} + ...) = \frac{\pi}{1+\frac{9}{16}} = \frac{16\pi}{25}$

**15** 10 cards labeled 1 through 10 lie on a table. Kevin randomly takes 3 cards and Patrick randomly takes 2 of the remaining 7 cards. What is the probability that Kevin's largest card is smaller than Patrick's largest card, and that Kevin's second-largest card is smaller than Patrick's smallest card?

*Proposed by Bradley Guo.*

**Answer:** $\boxed{\frac{1}{5}}$

**Solution:** For this to happen, Patrick must have drawn the highest card out of the 5 cards chosen and the second-highest card, or the highest card and the third-highest card. There are a total of 10 possible pairs of cards that Patrick could have chosen, so the probability is $\frac{2}{10} = \frac{1}{5}$.