

# MBMT Team Round – Bernoulli

April 16, 2023

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

**DO NOT TURN THE QUESTION SHEET IN!  
Use the official answer sheet.**

You are highly encouraged to work with your teammates on the problems in order to solve them.

- 1 **[4]** What is the sum of the first 5 positive integers?
  
- 2 **[4]** Bread picks a number  $n$ . He finds out that if he multiplies  $n$  by 23 and then subtracts 20, he gets 46279. What is  $n$ ?
  
- 3 **[4]** A Harshad Number is a number that is divisible by the sum of its digits. For example, 27 is divisible by  $2 + 7 = 9$ . Only one two-digit multiple of 9 is not a Harshad Number. What is this number?
  
- 4 **[5]** There are 5 red balls and 3 blue balls in a bag. Alice randomly picks a ball out of the bag and then puts it back in the bag. Bob then randomly picks a ball out of the bag. What is the probability that Alice gets a red ball and Bob gets a blue ball, assuming each ball is equally likely to be chosen?
  
- 5 **[5]** Let  $a$  be a 1-digit positive integer and  $b$  be a 3-digit positive integer. If the product of  $a$  and  $b$  is a 4-digit integer, what is the minimum possible value of the sum of  $a$  and  $b$ ?
  
- 6 **[5]** A circle has radius 6. A smaller circle with the same center has radius 5. What is the probability that a dart randomly placed inside the outer circle is outside the inner circle?
  
- 7 **[6]** Call a two-digit integer “sus” if its digits sum to 10. How many two-digit primes are sus?
  
- 8 **[6]** Alex and Jeff are playing against Max and Alan in a game of tractor with 2 standard decks of 52 cards. They take turns taking (and keeping) cards from the combined decks. At the end of the game, the 5s are worth 5 points, the 10s are worth 10 points, and the kings are worth 10 points. Given that a team needs 50 percent more points than the other to win, what is the minimal score Alan and Max need to win?

- 9 **[7]** Bob has a sandwich in the shape of a rectangular prism. It has side lengths 10, 5, and 5. He cuts the sandwich along the two diagonals of a face, resulting in four pieces. What is the volume of the largest piece?
- 10 **[8]** Aven makes a rectangular fence of area 96 with side lengths  $x$  and  $y$ . John makes a larger rectangular fence of area 186 with side lengths  $x + 3$  and  $y + 3$ . What is the value of  $x + y$ ?
- 11 **[8]** A number is prime if it is only divisible by itself and 1. What is the largest prime number  $n$  smaller than 1000 such that  $n + 2$  and  $n - 2$  are also prime? Note: 1 is not prime.
- 12 **[9]** Sally has 3 red socks, 1 green sock, 2 blue socks, and 4 purple socks. What is the probability she will choose a pair of matching socks when only choosing 2 socks without replacement?
- 13 **[9]** A triangle with vertices at  $(0, 0)$ ,  $(3, 0)$ ,  $(0, 6)$  is filled with as many  $1 \times 1$  lattice squares as possible. How much of the triangle's area is not filled in by the squares?
- 14 **[10]** A series of concentric circles  $w_1, w_2, w_3, \dots$  satisfy that the radius of  $w_1 = 1$  and the radius of  $w_n = \frac{3}{4}$  times the radius of  $w_{n-1}$ . The regions enclosed in  $w_{2n-1}$  but not in  $w_{2n}$  are shaded for all integers  $n > 0$ . What is the total area of the shaded regions?
- 15 **[10]** 10 cards labeled 1 through 10 lie on a table. Kevin randomly takes 3 cards and Patrick randomly takes 2 of the remaining 7 cards. What is the probability that Kevin's largest card is smaller than Patrick's largest card, and that Kevin's second-largest card is smaller than Patrick's smallest card?