1 Bob has 5 apples. What is the least number of apples he needs in addition to the apples he already has to be able to evenly divide his apples between himself and 2 friends?

Proposed by Daniel He and Evan Wang.

Answer: $\boxed{1}$

Solution: Adding one apple will allow him to give himself and his two friends two apples each. Thus, the minimum number of apples he needs is 1.

2 Lillian is thinking of a number, and Elina and Sophia must guess it correctly to win. As a hint, she tells them it's a positive square whose digits add up to 9. Elina guesses that the number is 36, but she is incorrect. Sophia guesses a lower number and wins. What number is Lillian thinking of?

Proposed by Elizabeth Yuan.

Answer: $\boxed{9}$

Solution: Because the digits add up to a multiple of 3, we know the number itself must be a multiple of 3. 3 is a prime number, so squares divisible by 3 must be divisible by 9 as well. The only square divisible by 9 and lower than 36 is 9.

3 Find the largest integer less than 2023 whose square ends in 9.

Proposed by Michelle Gao.

Answer: $\boxed{2017}$

Solution: Only numbers ending in 3 or 7 have squares ending in 9. The largest such integer less than 2023 is 2017.

4 How many positive integers divide both 100 and 160?

Proposed by Joshua Hsieh.

Answer: $\boxed{6}$

Solution: Any integer that divides both 100 and 160 divides their greatest common divisor, 20. There are 6 such numbers: $1, 2, 4, 5, 10, 20$.

5 How many integers less than 100 are divisible by 3 but not by 6?

Proposed by Evan Zhang.

Answer: $\boxed{17}$

Solution: There are 33 integers less than 100 that are divisible by 3 because $99 = 3 \cdot 33$. Similarly, there are 16 integers less than 100 that are divisible by 6 because $96 = 6 \cdot 16$. Since every multiple of 6 is a multiple of 3, to find the number of integers that are divisible by 3 but not 6, we can subtract to get a final answer of $33 - 16 = 17$

6 There exist positive integers a, b, c, with $b > 1$, and $6 \cdot a = b \cdot c = 12000$. If a and b are relatively prime, what is c?

Proposed by Isabelle Yang.

Answer: $\boxed{4000}$

Solution: We know that $a = \frac{12000}{6} = 2000$. By prime factorizing 12000, we have that $120 = 2^5 \cdot 3^1 \cdot 5^3$ We also know that a is relatively prime to b, so b must be relatively prime with $2000 = 2^4 \cdot 5^3$. Looking at the prime factors of 12000, we see that the only way a factor of 12000 can be relatively prime to 2000 is if it is 3 or 1. But $b > 1$ so $b = 3$. Thus, we get that $c = \frac{12000}{3} = 4000$.

7 What is the largest integer n such that 3^n is a factor of $18! + 19! + 20!$?

Proposed by Kian Dhawan.

Answer: $\boxed{8}$

Solution: We first notice that each of the terms is divisible by 18! so we can factor it out. This gives us $18! + 19! + 20! = 18!(1 + 19 + 19 \cdot 20) = 18! \cdot (400)$. 400 has no factors of 3, so we just count the factors of 3 in 18!. There is 1 factor of 3 from 3, 6, 12, 15, and 2 factors of 3 from 9, 18. This adds to a total of 8 factors of 3, thus the answer is 8.

8 For some positive integer $1 \leq n \leq 1000$, Jeremy writes down n^2, n^1, and n^0 in a row on his whiteboard, in that order. His friend Joshua, however, read the three integers as a single integer and deduced that it is a multiple of 3. For how many n would this happen?

Proposed by Kelin Zhu.

Answer: $\boxed{334}$

Solution: Notice that since $10^k \equiv 1 \pmod{3}$, the concatenation of n^2, n^1, n^0 is equivalent to $n^2 + n + 1$ mod 3. Testing $n = 0, 1, 2$ we find that this only works for $n \equiv 1$ mod 3. There are 334 possible solutions between 1 and 1000.