

MBMT Number Theory Round – Bernoulli

April 16, 2023

Full Name _____

Student ID Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- _____ 1 Bob has 5 apples. What is the least number of apples he needs in addition to the apples he already has to be able to evenly divide his apples between himself and 2 friends?
- _____ 2 Lillian is thinking of a number, and Elina and Sophia must guess it correctly to win. As a hint, she tells them it's a positive square whose digits add up to 9. Elina guesses that the number is 36, but she is incorrect. Sophia guesses a lower number and wins. What number is Lillian thinking of?
- _____ 3 Find the largest integer less than 2023 whose square ends in 9.
- _____ 4 How many positive integers divide both 100 and 160?
- _____ 5 How many integers less than 100 are divisible by 3 but not by 6?
- _____ 6 There exist positive integers a, b, c , with $b > 1$, and $6 \cdot a = b \cdot c = 12000$. If a and b are relatively prime, what is c ?
- _____ 7 What is the largest integer n such that 3^n is a factor of $18! + 19! + 20!$?
- _____ 8 For some positive integer $1 \leq n \leq 1000$, Jeremy writes down n^2, n^1 , and n^0 in a row on his whiteboard, in that order. His friend Joshua, however, read the three integers as a single integer and deduced that it is a multiple of 3. For how many n would this happen?