

Solutions to Bernoulli Geometry

- 1 If the values of two angles in a triangle are 60 and 75 degrees respectively, what is the measure of the third angle?

Proposed by Evan Zhang.

Answer: $\boxed{45}$

Solution: The sum of three angles in a triangle is 180 so the third angle's measure is $180 - 60 - 75 = 45$.

- 2 Square $ABCD$ has side length 1. What is the area of triangle ABC ?

Proposed by Bradley Guo.

Answer: $\boxed{\frac{1}{2}}$

Solution: Triangle ABC has the same area as triangle ACD and their sum is 1. So triangle ABC has area $\frac{1}{2}$.

- 3 An equilateral triangle and a square have the same perimeter. If the side length of the equilateral triangle is 8, what is the square's side length?

Proposed by Evan Zhang.

Answer: $\boxed{6}$

Solution: The perimeter of the triangle is $3 \cdot 8 = 24$. The perimeter of the square is also 24 so its side length must be $\frac{24}{4} = 6$.

- 4 What is the maximum possible number of sides and diagonals of equal length in a quadrilateral?

Proposed by Bradley Guo.

Answer: $\boxed{5}$

Solution: A rhombus with angles 60 and 120 degrees has 4 sides and 1 diagonal of equal length. A quadrilateral with all 4 sides and 2 diagonals of equal length is impossible because a rhombus with equal-length diagonals is a square, which has a diagonal that is longer than its side length.

- 5 A square of side length 4 is put within a circle such that all 4 corners lie on the circle. What is the diameter of the circle?

Proposed by Kevin Yao.

Answer: $\boxed{4\sqrt{2}}$

Solution: The diameter of the circle will be a diagonal of the square by symmetry. The side length of the square is by Pythagorean theorem $\sqrt{4^2 + 4^2} = 4\sqrt{2}$.

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- 6 Patrick is rafting directly across a river 20 meters across at a speed of 5 m/s. The river flows in a direction perpendicular to Patrick's direction at a rate of 12 m/s. When Patrick reaches the shore on the other end of the river, what is the total distance he has traveled?

Proposed by Kevin Yao.

Answer: $\boxed{52}$

Solution: Every second, Patrick moves 5 meters towards the other side of the river, and 12 meters along the river. Thus, he travels 13 meters every second for 4 seconds, which gives a total distance of 52.

- 7 Quadrilateral $ABCD$ has side lengths $AB = 7$, $BC = 15$, $CD = 20$, and $DA = 24$. It has a diagonal length of $BD = 25$. Find the measure, in degrees, of the sum of angles ABC and ADC .

Proposed by Kevin Yao.

Answer: $\boxed{180}$

Solution: 7-24-25 and 15-20-25 are both Pythagorean triples, and so two of the angles are right angles. Since the sum of the internal angles of a quadrilateral is 360, $m\angle ABC + m\angle ADC = 360 - 180 = 180$.

- 8 What is the largest P such that any rectangle inscribed in an equilateral triangle of side length 1 has a perimeter of at least P ?

Proposed by Bradley Guo.

Answer: $\boxed{\sqrt{3}}$

Solution: Two corners of the rectangle must be on the same side of the triangle. Let the side length of the edge between the two corners be x . Then the other side length of the rectangle is $\frac{(1-x)\sqrt{3}}{2}$, and so the perimeter of the rectangle is $\sqrt{3} + (2 - \sqrt{3})x$. This is minimized at $x = 0$ which gives an answer of $\sqrt{3}$.