

Solutions to Bernoulli Algebra

- 1 A car is driving at 60 miles an hour. How many miles will it travel in 5 hours?

Proposed by Evan Zhang.

Answer: $\boxed{300}$

Solution: Distance is equal to rate multiplied by time, so the answer is $60 \text{ mph} \cdot 5 \text{ hours} = 300 \text{ miles}$.

- 2 Mario has some fire flowers and some ice flowers. He has a total of 16 flowers, and he has 3 times more fire flowers than ice flowers. How many ice flowers does he have?

Proposed by Jeffrey Boman.

Answer: $\boxed{4}$

Solution: Let the number of ice flowers Mario have be equal to x . Then, Mario must then have $3x$ fire flowers. Summing these, we find that Mario has $4x$ total flowers. Since $4x$ must be 16, x must be 4, so Mario must have 4 ice flowers.

- 3 Yunyi discovers that when he multiplies a number by 2, he gets the same result as if he added 5 to the number. What is the number?

Proposed by Evan Zhang.

Answer: $\boxed{5}$

Solution: Let Yunyi's number be x . We are given that $2x = x + 5$. Subtracting x from both sides, we get $x = 5$.

- 4 At noon, a clock shows the correct time. At m minutes after noon, the clock suddenly starts ticking at half the speed it should. At 1:30, the clock shows 1 o'clock. Find the value of m .

Proposed by Ivy Guo.

Answer: $\boxed{30}$

Solution: After 90 minutes, the clock is 30 minutes behind. After the clock slows down, every two minutes, the clock gets one minute behind. It would take 60 minutes for the clock to get 30 minutes behind. Therefore, for the first 30 minutes, the clock must tick at the correct speed, so $m = 30$.

- 5 Victor is making a batch of brownies. He takes the pan and makes a number of cuts parallel to the edges of the pan. If he ends up with 63 brownies of equal size, what is the minimum number of cuts he could have made?

Proposed by Evan Zhang.

Answer: $\boxed{14}$

Solution: Note that if Victor wants to make x "strips", it will require $x - 1$ cuts. That means that if there are xy brownies, $(x - 1) + (y - 1) = x + y - 2$ cuts are required. We just want to minimize the sum $x + y - 2$, which is equivalent to minimizing $x + y$. We

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are given that $xy = 63$, and $x = 7, y = 9$ minimizes the sum. Thus, Victor can make a minimum of $7 + 9 - 2 = 14$ cuts.

- 6 Compute $\sqrt{10004 \cdot 9996 + 16}$.

Proposed by Michelle Gao.

Answer: $\boxed{10000}$

Solution: Notice that $10004 = 10000 + 4$ and $9996 = 10000 - 4$. We can then create a difference of squares: $\sqrt{(10000 + 4)(10000 - 4) + 16} = \sqrt{(10000^2 - 16) + 16} = 10000$.

- 7 Let a_n be the sum of integers from 1 to n (for instance, $a_1 = 1, a_3 = 1 + 2 + 3$). And let $b_n = \frac{a_{2n-1}}{a_{2n}}$. Find $b_1 \cdot b_2 \cdot b_3 \cdot \dots \cdot b_{10}$.

Proposed by Evan Wang.

Answer: $\boxed{\frac{1}{21}}$

Solution: The sum of number 1 to n is given by the formula $\frac{n(n+1)}{2}$. This means $b_n = \frac{\frac{(2n-1)2n}{2}}{\frac{2n(2n+1)}{2}} = \frac{2n-1}{2n+1}$. Thus, the product is equal to $\frac{1}{3} \cdot \frac{3}{5} \cdot \frac{5}{7} \cdot \dots \cdot \frac{19}{21}$ which simplifies to $\frac{1}{21}$.

- 8 Suppose Bradley has a sequence such that $x_n = 3x_{n-1} + 2$. If $x_0 = 0$, then what is x_{20} ?

Proposed by Kevin Wu.

Answer: $\boxed{3^{20} - 1}$

Solution: Let $y_n + c = x_n$, where c is a constant to be chosen later. Then, we get $y_n + c = 3y_{n-1} + 3c + 2$, or $y_n = 3y_{n-1} + 2c + 2$. Setting $c = -1$, we get $y_n = 3y_{n-1} + (-2) + 2 = 3y_{n-1}$. Because $x_0 = 0$, we know $y_0 = 1$, and since $y_{20} = y_{19} \cdot 3 = y_{18} \cdot 3^2 = \dots = y_0 \cdot 3^{20} = 3^{20}$, we can find that $x_{20} = 3^{20} - 1$.