

# MBMT Team Round – Zermelo

May 21, 2022

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

**DO NOT TURN THE QUESTION SHEET IN!  
Use the official answer sheet.**

You are highly encouraged to work with your teammates on the problems in order to solve them.

- 1 [4] What percent of the first 20 positive integers are divisible by 3?
  
- 2 [4] It is said that a sheet of printer paper can only be folded in half 7 times. A sheet of paper is 8.5 inches by 11 inches. What is the ratio of the paper's area after it has been folded in half 7 times to its original area?
  
- 3 [4] Boba has an integer. They multiply the number by 8, which results in a two digit integer. Bubbles multiplies the same original number by 9 and gets a three digit integer. What was the original number?
  
- 4 [5] For how many integers  $x$  is  $9x^2$  greater than  $x^4$ ?
  
- 5 [5] How many two digit numbers are the product of two distinct prime numbers ending in the same digit?
  
- 6 [5] A triangle's area is twice its perimeter. Each side length of the triangle is doubled, and the new triangle has area 60. What is the perimeter of the new triangle?
  
- 7 [6] Let  $F$  be a point inside regular pentagon  $ABCDE$  such that  $\triangle FDC$  is equilateral. Find  $\angle BEF$ .
  
- 8 [6] Carl, Max, Zach, and Amelia sit in a row with 5 seats. If Amelia insists on sitting next to the empty seat, how many ways can they be seated?
  
- 9 [7] The numbers 1, 2, ..., 29, 30 are written on a whiteboard. Gumbo circles a bunch of numbers such that for any two numbers he circles, the greatest common divisor of the two numbers is the same as the greatest common divisor of all the numbers he circled. Gabi then does the same. After this, what is the least possible number of uncircled numbers?

- 10 [8] Via has a bag of veggie straws, which come in three colors: yellow, orange, and green. The bag contains 8 veggie straws of each color. If she eats 22 veggie straws without considering their color, what is the probability she eats all of the yellow veggie straws?
- 11 [8] We call a string of letters *purple* if it is in the form  $CVCCCV$ , where  $C$ s are placeholders for (not necessarily distinct) consonants and  $V$ s are placeholders for (not necessarily distinct) vowels. If  $n$  is the number of *purple* strings, what is the remainder when  $n$  is divided by 35? The letter  $y$  is counted as a vowel.
- 12 [9] Let  $a, b, c,$  and  $d$  be integers such that  $a+b+c+d = 0$  and  $(a+b)(c+d)(ab+cd) = 28$ . Find  $abcd$ .
- 13 [9] Griffith is playing cards. A 13-card hand with Aces of all 4 suits is known as a *godhand*. If Griffith and 3 other players are dealt 13-card hands from a standard 52-card deck, then the probability that Griffith is dealt a *godhand* can be expressed in simplest form as  $\frac{a}{b}$ . Find  $a$ .
- 14 [10] For some positive integer  $m$ , the quadratic  $x^2 + 202200x + 2022m$  has two (not necessarily distinct) integer roots. How many possible values of  $m$  are there?
- 15 [10] Triangle  $ABC$  with altitudes of length 5, 6, and 7 is similar to triangle  $DEF$ . If  $\triangle DEF$  has integer side lengths, find the least possible value of its perimeter.