1 Point E is on side AB of rectangle $ABCD$. Find the area of triangle ECD divided by the area of rectangle $ABCD$.

Proposed by Nathan Cho.

Answer: $\boxed{\frac{1}{2}}$

Solution: Consider the height to the base CD of ECD. Since E is on AB, the height is simply the distance between AB and CD. Therefore, the area of the triangle is just half of the area of the rectangle.

2 Garb and Grunt have two rectangular pastures of area 30. Garb notices that his has a side length of 3, while Grunt's has a side length of 5. What's the positive difference between the perimeters of their pastures?

Proposed by Nathan Cho.

Answer: $\boxed{4}$

Solution: We know that the area of a rectangle is the the two side lengths multiplied together.

Garb's rectangle has area $30 = 3 \cdot x$, so $x = 10$, meaning that his rectangle has side lengths 3 and 10. The perimeter is then $3 + 3 + 10 + 10 = 26$.

For Grunt, we get $30 = 5 \cdot y$, so $y = 6$, and his rectangle has perimeter $5+5+6+6 = 22$.

The answer is $26 - 22 = 4$.

3 A scalene triangle (the 3 side lengths are all different) has integer angle measures (in degrees). What is the largest possible difference between two angles in the triangle?

Proposed by Bradley Guo.

Answer: $\boxed{176}$

Solution: A scalene triangle has three different angles all summing to 180. The largest possible difference between two of these angles happens in a triangle with angles $1°$, $2°$, and $177°$. The difference is $177 - 1 = 176$.

4 Let point E be on side $\overline{AB}$ of square $ABCD$ with side length 2. Given $DE = BC + BE$, find BE.

Proposed by Bradley Guo.

Answer: $\boxed{\frac{1}{2}}$

Solution: Let $BE = x$. Then $AE = 2 - x$ and $DE = 2 + x$. Applying the Pythagorean Theorem gives

$$2^2 + (2 - x)^2 = (2 + x)^2$$

Expanding and solving for x gives $x = \frac{1}{2}$

5 The two diagonals of rectangle $ABCD$ meet at point E. If $\angle AEB = 2\angle BEC$, and $BC = 1$, find the area of rectangle $ABCD$.

Proposed by Bradley Guo.

Answer: $\boxed{\sqrt{3}}$

Solution: We know that $\angle AEB + \angle BEC = 180°$ because E lies on AC, so then $3\angle BEC = 180°$ and $\angle BEC = 60°$. Therefore, $\angle DBC = 60°$, so since $\angle DCB = 90°$ we know that DBC is a 30-60-90 triangle, and $DC = \sqrt{3}$. Therefore, the area is $DC \cdot BC = \sqrt{3}$.

6 In $\triangle ABC$, let D be the foot of the altitude from A to BC. Additionally, let X be the intersection of the angle bisector of $\angle ACB$ and AD. If $BD = AC = 2AX = 6$, find the area of ABC.

Proposed by Nathan Cho.

Answer: $\boxed{\dfrac{576}{25}}$

Solution: We know by the angle bisector theorem in triangle ACD that $\frac{DC}{DX} = \frac{AC}{AX} = 2$. Letting $DX = x$, then $DC = 2x$. By Pythagorean Theorem in triangle ADC, we have $(2x)^2 + (3 + y)^2 = 36$. We can solve this to find that $x = \frac{9}{5}$. Then, the area of triangle ABC is $\frac{AD \cdot BC}{2} = \frac{(3+y)(6+2x)}{2} = \frac{576}{25}$.

7 Let $\triangle ABC$ have $\angle ABC = 40°$. Let D and E be on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{DE}$ is parallel to $\overline{BC}$, and the circle passing through points D, E, and C is tangent to $\overline{AB}$. If the center of the circle is O, find $\angle DOE$.

Proposed by Bradley Guo.

Answer: $\boxed{80°}$

Solution: We know that because $DE \parallel BC$, then $\angle ADE = \angle ABC = 40°$. Because the circle through DEC is tangent to $\overline{AB}$, then the measure for arc DE equals $\angle ADE = 40°$. Then, since $\angle DOE$ is the central angle corresponding to the arc, then the angle measure is twice the arc measure and we get $80°$.

8 Consider $\triangle ABC$ with $AB = 3$, $BC = 4$, and $AC = 5$. Let D be a point of AC other than A for which $BD = 3$, and E be a point on BC such that $\angle BDE = 90°$. Find EC.

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{7}{8}}$

Solution: We can angle chase to find

$$\angle C = 90 - \angle A = 90 - \angle ADB = 180 - \angle BDE - \angle ADB = \angle EDC$$

which implies that $\triangle DEC$ is iscoceles. Let $ED = EC = x$. Since $BE = 4 - x$, $BD = 3$, and $\angle BDE = 90$, we can apply Pythagorean Theorem on $\triangle BDE$:

$$3^2 + x^2 = (4 - x)^2$$

Expanding and solving for x gives $x = \frac{7}{8}$.