1 Two identical caps each have 4 paper slips with numbers 1, 2, 4, and 8 written on them. Madeline takes out one paper slip from each cap, multiplies the two numbers she sees, and buys that number of strawberries. How many different values are possible for the number of strawberries that she will buy?

Proposed by Bradley Guo and Justin Chen.

Answer: $\boxed{7}$

Solution: Notice that all these numbers are powers of 2, so the only products we can get are also powers of 2. The smallest product is 1 and the largest is 64, so the possible values are $1, 2, 4, 8, 16, 32, 64$ and there are 7 of them.

2 Two different positive integers sum to 10. How many possibilities are there for their product?

Proposed by Bradley Guo.

Answer: $\boxed{4}$

Solution: We let the minimum of the two different positive integers be a, then the other integer has to be $10 - a$ so the product will be $a(10 - a)$. Since the two integers are different, a cannot be 5. We know that a can range from 1 to 4 and each value of a gives a different product, thus there are 4 possibilities total.

3 A triangle has sides of length 2, a square has sides of length 3, and a pentagon has sides of length 4. Two sides are chosen from the 12 sides. What is the probability that the two chosen sides have the same length?

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{19}{66}}$

Solution: We can either have two sides from the triangle, two sides from the square, or two sides from the pentagon. The probability of the first case is $\frac{\binom{3}{2}}{\binom{12}{2}}$, the probability of the second is $\frac{\binom{4}{2}}{\binom{12}{2}}$, and the probability of the third is $\frac{\binom{5}{2}}{\binom{12}{2}}$. Adding gives $\frac{3+6+10}{66} = \frac{19}{66}$.

4 Gose shuffles a standard deck of 52 cards. He flips over the first three cards: 8, 2, and 3. What is the probability that when he flips over the fourth card, the sum of the four values is greater than 21? Jacks, queens, and kings have a value of 10, and aces have a value of 11.

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{24}{49}}$

Solution: For the sum of the four values to be greater than 21, then the fourth dice must be greater than $21 - 8 - 2 - 3 = 8$. Thus, it must be a $9, 10$, jack, queen, king, or ace. There are 4 of each of these rank of card in the deck, so there are 24 possible winning cards. The deck has 49 cards left since the first 3 were removed, so the probability is $\frac{24}{49}$.

5 Bread has a bag of two forks and three spoons. Squash reaches into the bag and pulls out three utensils. What is the probability that Squash has more spoons than forks?

Proposed by Nathan Cho.

Answer: $\boxed{\frac{7}{10}}$

Solution: For Squash to have more spoons, he can either have 2 or 3 spoons. There are $\binom{5}{3} = 10$ possible combinations Squash can pull. There is 1 way for Squash to pull 3 spoons, and $\binom{2}{1} \cdot \binom{3}{2} = 6$ ways for Squash to pull 2 spoons and 1 fork. This gives a probability of $\frac{6+1}{10} = \frac{7}{10}$.

6 Kevin initially has 4 coins atop a table. He flips all of them and puts each coin with heads facing up in his pocket. If any coins remain on the table, he repeats the procedure. What is the probability that, after these two steps, all 4 coins will be in Kevin's pocket?

Proposed by Kelin Zhu.

Answer: $\boxed{\frac{81}{256}}$

Solution: Notice that the game is the same if each coin is played separately, and then combined at the end. Thus, we can compute the probability that a given coin ends up in his pocket, and raise it to the fourth power. The probability that the coin ends up on the table is $\frac{1}{4}$ since it must land tails twice, so the probability it ends up in the pocket is $\frac{3}{4}$. Raising to the fourth power, we get the answer of $\frac{81}{256}$.

7 Two people play a game. They alternate rolling a die and each keeps a running sum of all the numbers they've rolled. The first person with a positive multiple of 3 wins. What is the probability that the first person wins?

Proposed by Joshua Hsieh.

Answer: $\boxed{\frac{3}{5}}$

Solution: Let P be the probability the first person wins. At any given point in the game, the person rolling has a $\frac{1}{3}$ chance of getting a positive multiple of 3 on the next roll, since there are two numbers on the dice for each remainder modulo 3. We use recursion. The first person going can either win on the first turn, or the game restarts with the roles changed. In the second case, because the roles are switched, the probability the original first person wins is $1 - P$. Thus, we have $P = \frac{1}{3} + \frac{2}{3}(1 - P)$, and solving gives $P = \frac{3}{5}$.

8 Knights only tell the truth, and Knaves only tell lies. 7 people that are each either Knights or Knaves line up in a line. Everyone except the person at the very right knows whether the person to their right is a Knight or a Knave. If 3 people say that the person to their right is a Knight, and 3 people say that the person to their right is a Knave, how many possible orderings of Knights and Knaves are there?

Proposed by Grace Hu.

Answer: $\boxed{40}$

Solution: If a Knight has a Knight to their right or if a Knave has a Knave to their right, they will say that the person to their right is a Knight. If a Knight has a Knave to their right or if a Knave has a Knight to their right, they will say that the person to their right is a Knight. Thus, whenever a person says Knave, we know that a change has been made, meaning that they are a different kind than the person to their right. With 7 people in a line, there are 6 opportunities for changes, and since 3 people said Knave, there are $\binom{6}{3} = 20$ possible orderings of changes. As the person on the very right can be either a Knight or a Knave, multiplying by 2 produces the final answer of 40.

Solution 2 We claim that for each distribution of the 6 statements, there are exactly two possible orderings of Knights and Knaves: one where the leftmost person is a knight, and one when they're a knave. To see this, notice that once we know the leftmost person and whether they say knight or knave, it determines the second leftmost, and so on, until the rightmost person, and because there is nothing said about the first person then they can be either knight or knave. There are $\binom{6}{3} = 20$ ways to order the statements, and 2 ways to choose the first person's type, so there are $20 \cdot 2 = 40$ orderings possible.