1 Michelle finds a website which gives her a free textbook every day. After gaining 10 textbooks in 10 days, she has tripled her textbook collection. How many textbooks does Michelle have 20 days after she found the website?

Proposed by Nathan Shan.

Answer: $\boxed{25}$

Solution: If Michelle starts with x textbooks, then we have that $x + 10 = 3x$, and moving the x to one side gives us that $2x = 10$, or equivalently, that $x = 5$. Therefore, in 20 days, Michelle will have $x + 20 = 25$ books.

2 Steven really likes palindromes. Palindromes are numbers that read the same backwards and forwards, like 55 or 969. He's thinking of a 3 digit palindrome where the sum of digits is 16 and the ones digit is 5 more than the tens digit. What is the number?

Proposed by Steven Wang.

Answer: $\boxed{727}$

Solution: The palindrome must have the same hundreds digit and units digit. We can write this as $\overline{aba} = 100a + 10b + a$, where a and b are digits. The problem tells us that $a + b + a = 16$ and $a = b + 5$. Substituting and collecting like terms, we get $3b + 10 = 6$, or $b = 2$.

Finally, we remember that $a = b + 5$, so $a = 7$.

3 Gablin and Babblin start with different amounts of grapes. If Gablin gives Babblin 1 grape, Gablin would have the number of grapes Babblin has, squared. If Babblin gave Gablin 1 grape instead, Gablin would have had the number of grapes Babblin has, cubed. How many grapes does Gablin have?

Proposed by Bradley Guo.

Answer: $\boxed{26}$

Solution: Let the number of grapes that Gablin has be x, and the number of grapes Babblin has be y.

The problem tells us that $x - 1 = (y + 1)^2$, and that $x + 1 = (y - 1)^3$.

Since $(y - 1)^3$ grows much faster than $(y + 1)^2$, we know that y is small. We find $y = 4$ by checking the first few values of y, and thus, $x = 26$.

4 Find the number of integers whose nearest perfect square is 264^2, including 264^2 itself.

Proposed by Bradley Guo.

Answer: $\boxed{528}$

Solution: There are $264^2 - 263^2 - 1 = (264 - 263)(264 + 263) - 1 = 526$ numbers between 264^2 and 263^2, exclusive.

There are $265^2 - 264^2 - 1 = (265 - 264)(265 + 264) - 1 = 528$ numbers between 264^2 and 265^2, exclusive.

For each of the two groups of numbers, exactly half of them are closer to 264^2.

Including 264^2, we get a final answer of $\frac{526}{2} + \frac{528}{2} + 1 = 528$.

5 What fraction of real numbers between 0 and 2 satisfy $\lceil x - \frac{2}{15} \rceil = \lfloor x + \frac{2}{15} \rfloor$? Here, $\lceil x \rceil$ is the smallest integer greater than or equal to x, and $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{4}{15}}$

Solution: We let $\{x\} = x - \lfloor x \rfloor$ be the fractional part of x. We can split this into 3 cases: If $\{x\} \leq \frac{2}{15}$, then $x - \frac{2}{15} < \lfloor x \rfloor$, so $\left\lceil x - \frac{2}{15} \right\rceil = \lfloor x \rfloor = \left\lfloor x + \frac{2}{15} \right\rfloor$.

If $\frac{2}{15} < \{x\} < \frac{13}{15}$, then $\left\lceil x - \frac{2}{15} \right\rceil = \lfloor x \rfloor + 1$, but $\left\lfloor x + \frac{2}{15} \right\rfloor = \lfloor x \rfloor$.

If $\frac{13}{15} \leq \{x\}$, then $\left\lceil x - \frac{2}{15} \right\rceil = \lfloor x \rfloor + 1 = \left\lfloor x + \frac{2}{15} \right\rfloor$.

Thus we know that the values of x that work are $\left[0, \frac{2}{15}\right] \cup \left[\frac{13}{15}, \frac{17}{15}\right] \cup \left[\frac{28}{15}, 2\right]$, giving a fraction of $\frac{4}{15}$.

6 For some constant value(s) of c, the following system of equations has infinite solutions.

$$\begin{cases} x = |y - 4| - 20 \\ y = |x - c| \end{cases}$$

What is the sum of all possible values of c?

Proposed by Nathan Shan.

Answer: $\boxed{-40}$

Solution: We graph the top equation to see that the solution set is composed of two rays, both with vertex $(-20, 4)$ and one ray with equation $x = y - 24$ and one with $x = -y - 16$. The bottom equation will also have two rays, one with positive slope and one with negative slope, so for the two equations combined to have infinitely many solutions, at least one pair of rays must overlap and have the same equation.

The two rays for the bottom equation will be $y = x - c$ and $y = c - x$, so either $x = y - 24$ and $y = x - c$ overlap implying $c = -24$, or $x = -y - 16$ and $y = c - x$ overlap implying $c = -16$, and the sum of the two is -40.

7 Given real x, y, and z such that $x + y = 4$ and $x^2 + y^2 + xy + xz + yz = 288$, find the maximum possible value of z.

Proposed by Bradley Guo.

Answer: $\boxed{69}$

Solution: We can rewrite $x^2 + y^2 + xy + xz + yz = 288$ as

$$(x + y)^2 - xy + (x + y)z = 16 - xy + 4z = 288.$$

xy is largest when $x = y = 2$, by AM-GM. This gives $z = \boxed{69}$.

8 Let $x_0 = 4$ and $x_{n+1} = x_n + 2\sqrt{x_n - 1} + 1$ for $n \geq 0$. What is x_{100}?

Proposed by Clarence Lam.

Answer: $\boxed{10004 + 200\sqrt{3}}$

Solution: Define the sequence y_n such that $y_n = \sqrt{x_n - 1}$; then $y_0 = \sqrt{3}$ and $y_{n+1} = y_n + 1$. Thus $y_n = n + \sqrt{3}$ and $x_n = (n + \sqrt{3})^2 + 1$. Plug in $n = 100$ and expand to get the final answer.