1 The product of two positive integers is 5. What is their sum?

Proposed by Bradley Guo.

Answer: $\boxed{6}$

Solution: As 5 is prime, one of the integers must be 5 while the other must be 1. Thus, the sum is $5+1 = 6$.

2 Gavin is 4 feet tall. He walks 5 feet before falling forward onto a cushion. How many feet is the top of Gavin's head from his starting point?

Proposed by Bradley Guo.

Answer: $\boxed{9}$

Solution: The distance from the starting point to Gavin's feet is 5 feet, and the distance from Gavin's feet to his head is 4 feet. Since Gavin feel forward onto the floor, he is lying flat, meaning that the line connecting Gavin's head and the starting point also contains Gavin's feet. Thus, the distance from the starting point to Gavin's head is $5+4 = 9$ feet.

3 How many times must Nathan roll a fair 6-sided die until he can guarantee that the sum of his rolls is greater than 6?

Proposed by Bradley Guo.

Answer: $\boxed{7}$

Solution: If Nathan rolls a 1 6 times, the sum of his rolls is 6, which is not greater than 6. However, if he rolls 7 times, even if he gets a 1 every single time, his total sum will be 7, so he must roll the die 7 times to guarentee that the sum of his rolls are greater than 6.

4 What percent of the first 20 positive integers are divisible by 3?

Proposed by Bradley Guo.

Answer: $\boxed{30}$

Solution: A third of integers are divisible by 3. However, only 6 integers within the first 20 positive integers are divisible by 3, so the answer is $\frac{6}{20} = 30\%$.

5 Let a be a positive integer such that $a^2 + 2a + 1 = 36$. Find a.

Proposed by Bradley Guo.

Answer: $\boxed{5}$

Solution: Notice that $a^2 + 2a + 1 = (a + 1)^2$, so we have $(a + 1)^2 = 36$ so $a + 1 = \pm 6$, so $a = 5$ or $a = -7$. Since -7 is not a positive integer, a must be 5.

6 It is said that a sheet of printer paper can only be folded in half 7 times. A sheet of paper is 8.5 inches by 11 inches. What is the ratio of the paper's area after it has been folded in half 7 times to its original area?

Proposed by River Chen.

Answer: $\boxed{\dfrac{1}{128}}$

Solution: Folding a paper halves one dimension, therefore halving the area. That means that seven folds halves the area 7 times, so the ratio of the paper's area after it has been folded in half 7 times to its original area is $\frac{(\frac{1}{2})^7}{1} = \frac{1}{128}$

7 Boba has an integer. They multiply the number by 8, which results in a two digit integer. Bubbles multiplies the same original number by 9 and gets a three digit integer. What was the original number?

Proposed by Bradley Guo.

Answer: $\boxed{12}$

Solution: Because multiplying the number by 8 results in a two digit number, the number must be between 2 and 12, inclusive. Furthermore, because multiplying the number by 9 results in a three digit integer, the number must be at least 12. Thus, the only number the original number could be is 12.

8 The average number of letters in the first names of students in your class of 24 is 7. If your teacher, whose first name is Blair, is also included, what is the new class average?

Proposed by Elina Lee.

Answer: $\boxed{6.92 \text{ or } \dfrac{173}{25}}$

Solution: The number of letters in the first names of the students in the class is $24 \cdot 7 = 168$. Adding the teacher's name results in 173 letters, and dividing that by the total number of people in the class gives $\frac{173}{25} = 6.92$ letters.

9 For how many integers x is $9x^2$ greater than x^4?

Proposed by Bradley Guo.

Answer: $\boxed{4}$

Solution: If $9x^2 > x^4$, we can divide x^2 (since x^2 can't be negative, we can divide it without needing to flip the inequality sign) to get $9 > x^2$. Thus, x has to be between -3 and 3, not inclusive, so there are only five integers it can be. However, if x is 0, we never could have divided by x^2, and it turns out that $9x^2$ is equal to x^4 when x is 0. That means the only four integers that work are $-2, -1, 1,$ and 2.

10 How many two digit numbers are the product of two distinct prime numbers ending in the same digit?

Proposed by Kevin Yao.

Answer: $\boxed{2}$

Solution: We know that the two numbers cannot both be larger than 10, as then their product would be at least three digits. That means that one of the numbers has to be an one digit prime. Looking at the one digit primes, we can see that if the prime is 3, we can multiply it by 13 or 23 to form a two digit number, but 33 isn't prime and 43 makes the product larger than 99. If the prime is 5, no number ending in 5 greater than 5 can't be prime. If the prime is 7, the first prime number ending in the same digit larger than that prime is 17, but that product is larger than 100. As there are no other one digit primes, the only products possible are $3 \cdot 13 = 39$ and $3 \cdot 23 = 69$.

11 A triangle's area is twice its perimeter. Each side length of the triangle is doubled, and the new triangle has area 60. What is the perimeter of the new triangle?

Proposed by Bradley Guo.

Answer: $\boxed{15}$

Solution: Since the new triangle has area 60, the old triangle must have had area $\frac{60}{4} = 15$. That tells us that the old triangle had perimeter $\frac{15}{2}$, so the new one must have perimeter $\frac{15}{2} \cdot 2 = 15$

12 Let F be a point inside regular pentagon $ABCDE$ such that $\triangle FDC$ is equilateral. Find $\angle BEF$.

Proposed by Kevin Wu.

Answer: $\boxed{6}$

Solution: The angles in a regular pentagon are all $108°$. Thus, $\angle EDF = \angle BCF = 108 - 60 = 48°$. Since $\triangle$ FDC is equilateral, $\overline{DC} = \overline{DF} = \overline{FC}$. That means that $\overline{ED} = \overline{FD}$, so $\triangle EDF$ is isoceles. This tells us that $\angle EFD = \frac{180-48}{2} = 66$. Then, $\angle EFB = 360 - 60 - 66 - 66 = 168$. Due to symmetry, $\overline{EF} = \overline{FB}$, so $\angle BEF = \frac{180-168}{2} = 6°$.

13 Carl, Max, Zach, and Amelia sit in a row with 5 seats. If Amelia insists on sitting next to the empty seat, how many ways can they be seated?

Proposed by Bradley Guo.

Answer: $\boxed{48}$

Solution: Notice that Amelia and the chair must always be together, so we can think of Amelia and the empty chair as one being, Chairmelia. If we seat Chairmelia, Carl, Max, and Zach, there are $4! = 24$ ways to arrange them. However, the arrangement of the chair and Amelia can happen in two ways, with the the chair being left of Amelia or right of Amelia. Thus, there are $24 \cdot 2 = 48$ ways.

14 The numbers $1, 2, ..., 29, 30$ are written on a whiteboard. Gumbo circles a bunch of numbers such that for any two numbers he circles, the greatest common divisor of the two numbers is the same as the greatest common divisor of all the numbers he circled. Gabi then does the same. After this, what is the least possible number of uncircled numbers?

Proposed by Nathan Cho.

Answer: $\boxed{12}$

Solution: Call a set S *good* if the pairwise gcds of numbers in S is constant.

We note that there are 10 prime numbers that are less than or equal to 30.

Claim If there are k distinct prime factors appearing in the prime factorizations of all numbers in a set S, then any subset $A \subset S$ with $|A| > k+1$, then A cannot be *good*.

Proof. By assumption, A is good with greatest common divisor c. Then, let

$$B = \left\{ \frac{a}{c} \mid a \in A \right\}.$$

Since we are dividing out by the greatest common divisor, any two numbers in B must be coprime.

Let prime p appearing as a divisor in set S. Then, note that if p appears in the factorizations of multiple numbers in B, then clearly those numbers are not coprime. Hence, it follows that p can only divide one number in B.

Since at most one number in B can be 1, and no numbers in B can be equal, we see that choosing $k+2$ numbers would imply that at least $k+1$ numbers of B are not equal to 1. But by pigeonhole principle, this would mean that there would have to be two numbers that are divisible by the same prime, which is contradiction. □

By our claim, Gumbo can choose at most 11 numbers. If we remove these numbers from S, we see that we are left with at least 6 prime numbers, since we can at most remove one occurrence of a number divisible by any given prime, and only 4 primes occur exactly once in our original S (namely, 17, 19, 23, 29).

It follows from our claim once more that Gabi can choose at most 7 numbers from the new S. Hence, the two can choose at most 18 distinct numbers, which we will show by example must be possible:

Gumbo can choose the numbers

$$1, 3, 5, 7, 11, 13, 16, 17, 19, 23, 29,$$

while Gabi chooses the numbers

$$2, 4, 6, 10, 14, 22, 26.$$

Thus, the answer is $30 - 18 = \boxed{12}$.

15 Via has a bag of veggie straws, which come in three colors: yellow, orange, and green. The bag contains 8 veggie straws of each color. If she eats 22 veggie straws without considering their color, what is the probability she eats all of the yellow veggie straws?

Proposed by Bradley Guo.

Answer: $\boxed{\dfrac{10}{23}}$

Solution: Let us instead consider the straws that she does not eat. She has two straws remaining at the end, if she eats all yellow straws, none of them can be yellow. There are $\binom{24}{2}$ ways to choose the color of both straws, and only $\binom{16}{2}$ for neither to be yellow, so the answer is $\frac{10}{23}$