MBMT Team Round — Dedekind

May 21, 2022

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

1 **[4]** The product of two positive integers is 5. What is their sum?

2 **[4]** Gavin is 4 feet tall. He walks 5 feet before falling forward onto a cushion. How many feet is the top of Gavin's head from his starting point?

3 **[4]** How many times must Nathan roll a fair 6-sided die until he can guarantee that the sum of his rolls is greater than 6?

4 **[5]** What percent of the first 20 positive integers are divisible by 3?

5 **[5]** Let a be a positive integer such that $a^2 + 2a + 1 = 36$. Find a.

6 **[5]** It is said that a sheet of printer paper can only be folded in half 7 times. A sheet of paper is 8.5 inches by 11 inches. What is the ratio of the paper's area after it has been folded in half 7 times to its original area?

7 **[6]** Boba has an integer. They multiply the number by 8, which results in a two digit integer. Bubbles multiplies the same original number by 9 and gets a three digit integer. What was the original number?

8 **[6]** The average number of letters in the first names of students in your class of 24 is 7. If your teacher, whose first name is Blair, is also included, what is the new class average?

9 **[7]** For how many integers x is $9x^2$ greater than x^4?

10 **[8]** How many two digit numbers are the product of two distinct prime numbers ending in the same digit?

11 **[8]** A triangle's area is twice its perimeter. Each side length of the triangle is doubled, and the new triangle has area 60. What is the perimeter of the new triangle?

12 **[9]** Let F be a point inside regular pentagon $ABCDE$ such that $\triangle FDC$ is equilateral. Find $\angle BEF$.

13 **[9]** Carl, Max, Zach, and Amelia sit in a row with 5 seats. If Amelia insists on sitting next to the empty seat, how many ways can they be seated?

14 **[10]** The numbers $1, 2, ..., 29, 30$ are written on a whiteboard. Gumbo circles a bunch of numbers such that for any two numbers he circles, the greatest common divisor of the two numbers is the same as the greatest common divisor of all the numbers he circled. Gabi then does the same. After this, what is the least possible number of uncircled numbers?

15 **[10]** Via has a bag of veggie straws, which come in three colors: yellow, orange, and green. The bag contains 8 veggie straws of each color. If she eats 22 veggie straws without considering their color, what is the probability she eats all of the yellow veggie straws?