1 How many positive integers divide 16?

Proposed by Nathan Shan.

Answer: $\boxed{5}$

Solution: We can factor $16 = 2^4$, so the only positive integers that work are $1, 2, 2^2, 2^3, 2^4$, giving our answer of 5.

2 Shron likes herding sheep. When he grouped his 47 sheep into groups of 9, he had some sheep remaining. How many sheep were remaining?

Proposed by Jason Youm.

Answer: $\boxed{2}$

Solution: We can write $47 = 9 \cdot 5 + 2$, so once the 5 groups are removed only 2 sheep remain.

3 What is the largest integer less than 100 that is not divisible by 2, 3, or 5?

Proposed by Bradley Guo.

Answer: $\boxed{97}$

Solution: We see that $99, 98$ are divisible by $3, 2$ respectively, but 97 works since it is prime.

4 Find the largest three digit integer which has an odd sum of digits, and an even product of digits.

Proposed by Bradley Guo.

Answer: $\boxed{988}$

Solution: Because the sum of digits is odd, then an odd number of digits are odd. Because the product is even, at least one digit is even, so at most two digits are odd. Thus, the number of odd digits is exactly 1. Since the largest digit 9 is odd and we want the largest such three digit number, the first digit must be 9 and the last two both 8.

5 Gabi has 5 consecutive positive integers. 3 of them are even, 2 are divisible by 3, one is divisible by 11. Find the smallest possible sum of the 5 integers.

Proposed by Nathan Cho.

Answer: $\boxed{50}$

Solution: The 5 integers are $8, 9, 10, 11, 12$. Notice that for 5 consecutive integers to contain 3 even integers, then the first integer must be even. To minimize the sum, we also want to have 11 as our value divisible by 11, since if one of the digits is 22 or higher the sum is clearly greater. Since 11 is odd, it must either be the second or forth number. Out of these two, only the sequence $8, 9, 10, 11, 12$ works, so it has the minimal sum.

6 How many zeros does $5! + 10! + 15! + 20! + 25!$ end in? Recall that $n! = 1 \cdot 2 \cdot \cdots \cdot n$.

Proposed by Bradley Guo.

Answer: $\boxed{1}$

Solution: First, we can notice that $5! = 120$ is divisible by 10 exactly once. Also, $10!, 15!, 20!, 25!$ are all divisible by 100 since in the product $1 \cdot 2 \cdot \cdots \cdot n$ we have both a $10, 5$, and a 2, so the last two digits are both 0. Thus, when we add them together, the last two digits will just be 20, so there is only 1 zero at the end of the sum.

7 An arithmetic sequence describes a list of numbers where each term is made by adding the same value, called the common difference. For example, the sequence $1, 3, 5$ has common difference 2, because each term is 2 greater than the last.

Kwu has 3 positive integers which form an arithmetic sequence with common difference 15. He multiplies the 3 integers and notices that the product is divisible by 120. He then adds the 3 integers. What is the minimum possible value of this sum?

Proposed by Bradley Guo.

Answer: $\boxed{135}$

Solution: Notice that since $3, 5$ divide the common difference, then they divide either all or none of the terms of the sequence. Since both divide 120, the common product, then they have to divide at least one term and thus divide all the terms. Therefore, we have that the sequence is of the form $15n, 15n + 15, 15n + 30$, which will have product $15n(15)(n+1)(15)(n+2)$. We can rewrite this as $15^3 n(n+1)(n+2)$. We need $120 = 15 \cdot 8$ to divide this, so then 8 has to divide $n(n+1)(n+2)$. The smallest n for which this works is $n = 2$, giving a minimal possible sum of $30 + 45 + 60 = 135$.

8 Suppose a, b, and c are equal to $2, 3$ and 4, in some order. What's the last digit of the greatest possible value of a^{b^c}?

Proposed by Bradley Guo.

Answer: $\boxed{2}$

Solution: We claim the greatest value is 2^{3^4}. To see this, we can split into cases based on what a is. If a is 2, then $3^4 = 81 > 4^3 = 64$, so then the maximum in that case is 2^{81}. If a is 3, then $2^4 = 4^2 = 16$, so we can just consider 3^{16}. If a is 4, then $3^2 = 9 > 2^3 = 8$, so we only consider 4^9. Finally, we can see that $2^{81} > 4^9 = 2^{18}$, and that $2^{81} = 8^{27} > 3^{16}$, so in the end 2^{81} is the largest.

To compute the units digit of 2^{81}, we can note that the sequence of units digits of $2, 2^2, 2^3, \ldots$ is $2, 4, 8, 6, 2, 4, 8, 6, \ldots$, so continuing the pattern we have 2^{81} must have units digit 2.