MBMT Number Theory Round — Dedekind

May 21, 2022

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** How many positive integers divide 16?

_____ **2** Shron likes herding sheep. When he grouped his 47 sheep into groups of 9, he had some sheep remaining. How many sheep were remaining?

_____ **3** What is the largest integer less than 100 that is not divisible by 2, 3, or 5?

_____ **4** Find the largest three digit integer which has an odd sum of digits, and an even product of digits.

_____ **5** Gabi has 5 consecutive positive integers. 3 of them are even, 2 are divisible by 3, one is divisible by 11. Find the smallest possible sum of the 5 integers.

_____ **6** How many zeros does $5! + 10! + 15! + 20! + 25!$ end in? Recall that $n! = 1 \cdot 2 \cdot \cdots \cdot n$.

_____ **7** An arithmetic sequence describes a list of numbers where each term is made by adding the same value, called the common difference. For example, the sequence $1, 3, 5$ has common difference 2, because each term is 2 greater than the last.

Kwu has 3 positive integers which form an arithmetic sequence with common difference 15. He multiplies the 3 integers and notices that the product is divisible by 120. He then adds the 3 integers. What is the minimum possible value of this sum?

_____ **8** Suppose a, b, and c are equal to $2, 3$ and 4, in some order. What's the last digit of the greatest possible value of a^{b^c}?