MBMT Dedekind Guts Round — Set 1

May 21, 2022

1 **[3]** What is $1 + 2 \cdot 3$?

2 **[3]** What is the average of the first 9 positive integers?

3 **[3]** A square of side length 2 is cut into 4 congruent squares. What is the perimeter of one of the 4 squares?

4 **[3]** Find the ratio of a circle's circumference squared to the area of the circle.

5 **[3]** 6 people split a bag of cookies such that they each get 21 cookies. Kyle comes and demands his share of cookies. If the 7 people then re-split the cookies equally, how many cookies does Kyle get?

MBMT Dedekind Guts Round — Set 2

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6 **[4]** How many prime numbers are perfect squares?

7 **[4]** Josh has an unfair 4-sided die numbered 1 through 4. The probability it lands on an even number is twice the probability it lands on an odd number. What is the probability it lands on either 1 or 3?

8 **[4]** If Alice consumes 1000 calories every day and burns 500 every night, how many days will it take for her to first reach a net gain of 5000 calories?

9 **[4]** Blobby flips 4 coins. What is the probability he sees at least one heads and one tails?

10 **[4]** Lillian has n jars and 48 marbles. If George steals one jar from Lillian, she can fill each jar with 8 marbles. If George steals 3 jars, Lillian can fill each jar to maximum capacity. How many marbles can each jar fill?

MBMT Dedekind Guts Round — Set 3

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11 **[5]** How many perfect squares less than 100 are odd?

12 **[5]** Jash and Nash wash cars for cash. Jash gets \$6 for each car, while Nash gets \$11 per car. If Nash has earned \$1 more than Jash, what is the least amount of money that Nash could have earned?

13 **[5]** The product of 10 consecutive positive integers ends in 3 zeros. What is the minimum possible value of the smallest of the 10 integers?

14 **[5]** Guuce continually rolls a fair 6-sided dice until he rolls a 1 or a 6. He wins if he rolls a 6, and loses if he rolls a 1. What is the probability that Guuce wins?

15 **[5]** The perimeter and area of a square with integer side lengths are both three digit integers. How many possible values are there for the side length of the square?

MBMT Dedekind Guts Round — Set 4

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16 **[7]** The cooking club at Blair creates 14 croissants and 21 danishes. Daniel chooses pastries randomly, stopping when he gets at least one croissant and at least two danishes. How many pastries must he choose to guarantee that he has one croissant and two danishes?

17 **[7]** Each digit in a 3 digit integer is either 1, 2, or 4 with equal probability. What is the probability that the hundreds digit is greater than the sum of the tens digit and the ones digit?

18 **[7]** How many two digit numbers are there such that the product of their digits is prime?

19 **[7]** In the coordinate plane, a point is selected in the rectangle defined by $-6 \leq x \leq 4$ and $-2 \leq y \leq 8$. What is the largest possible distance between the point and the origin, $(0, 0)$?

20 **[7]** The sum of two numbers is 6 and the sum of their squares is 32. Find the product of the two numbers.

MBMT Dedekind Guts Round — Set 5

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21 **[9]** Triangle ABC has area 4 and $\overline{AB} = 4$. What is the maximum possible value of $\angle ACB$?

22 **[9]** Let $ABCD$ be an iscoceles trapezoid with $AB = CD$ and M be the midpoint of $\overline{AD}$. If $\triangle ABM$ and $\triangle MCD$ are equilateral, and $BC = 4$, find the area of trapezoid $ABCD$.

23 **[9]** Let x and y be positive real numbers that satisfy $(x^2 + y^2)^2 = y^2$. Find the maximum possible value of x.

24 **[9]** In parallelogram $ABCD$, $\angle A \cdot \angle C - \angle B \cdot \angle D = 720^\circ$ where all angles are in degrees. Find the value of $\angle C$.

25 **[9]** The number $12ab9876543$ is divisible by 101, where a, b represent digits between 0 and 9. What is $10a + b$?

MBMT Dedekind Guts Round — Set 6

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26 **[12]** For every person who wrote a problem that appeared on the final MBMT tests, take the number of problems they wrote, and then take that number's factorial, and finally multiply all these together to get n. Estimate the greatest integer a such that 2^a evenly divides n.

27 **[12]** Circles of radius 5 are centered at each corner of a square with side length 6. If a random point P is chosen randomly inside the square, what is the probability that P lies within all four circles?

28 **[12]** Mr. Rose's evil cousin, Mr. Caulem, has teaches a class of three hundred bees. Every week, he tries to disrupt Mr. Rose's 4th period by sending three of his bee students to fly around and make human students panic. Unfortunately, no pair of bees can fly together twice, as then Mr. Rose will become suspicious and trace them back to Mr. Caulem. What's the largest number of weeks Mr. Caulem can disrupt Mr. Rose's class?

29 **[12]** Two blind brothers Beard and Bored are driving their tractors in the middle of a field facing north, and both are 10 meters west from a roast turkey. Beard, can turn exactly 0.7° and Bored can turn exactly 0.2° degrees. Driving at a consistent 2 meters per second, they drive straight until they notice the smell of the turkey getting farther away, and then turn right and repeat until they get to the turkey.

Suppose Beard gets to the Turkey in about 818.5 seconds. Estimate the amount of time it will take Bored.

30 **[12]** Let a be the probability that 4 randomly chosen positive integers have no common divisor except for 1. Estimate $300a$. Note that the integers $1, 2, 3, 4$ have no common divisor except for 1.

Remark. This problem is asking you to find

$$300 \lim_{n\to\infty} a_n,$$

if a_n is defined to be the probability that 4 randomly chosen integers from $\{1, 2, ..., n\}$ have greatest common divisor 1.