

## Solutions to Dedekind Geometry

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- 1 A Giant Hopper is 200 meters away from you. It can hop 50 meters. How many hops would it take for it to reach you?

*Proposed by Bradley Guo.*

**Answer:**  $\boxed{4}$

**Solution:** Each hop decreases the distance between you and the Giant Hopper by 50 meters. 4 jumps would decrease the distance by 200.

- 2 A rope of length 6 is used to form the edges of an equilateral triangle (a triangle with equal side lengths). What is the length of one of these edges?

*Proposed by Bradley Guo.*

**Answer:**  $\boxed{2}$

**Solution:** An equilateral triangle has three sides of equal length, meaning the total amount of rope used is triple the side length of the triangle. The length of one side is  $\frac{6}{3} = 2$ .

- 3 Point  $E$  is on side  $AB$  of rectangle  $ABCD$ . Find the area of triangle  $ECD$  divided by the area of rectangle  $ABCD$ .

*Proposed by Nathan Cho.*

**Answer:**  $\boxed{\frac{1}{2}}$

**Solution:** Consider the height to the base  $CD$  of  $ECD$ . Since  $E$  is on  $AB$ , the height is simply the distance between  $AB$  and  $CD$ . Therefore, the area of the triangle is just half of the area of the rectangle.

- 4 Garb and Grunt have two rectangular pastures of area 30. Garb notices that his has a side length of 3, while Grunt's has a side length of 5. What's the positive difference between the perimeters of their pastures?

*Proposed by Nathan Cho.*

**Answer:**  $\boxed{4}$

**Solution:** We know that the area of a rectangle is the the two side lengths multiplied together.

Garb's rectangle has area  $30 = 3 \cdot x$ , so  $x = 10$ , meaning that his rectangle has side lengths 3 and 10. The perimeter is then  $3 + 3 + 10 + 10 = 26$ .

For Grunt, we get  $30 = 5 \cdot y$ , so  $y = 6$ , and his rectangle has perimeter  $5 + 5 + 6 + 6 = 22$ .

The answer is  $26 - 22 = 4$ .

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- 5 Let points  $A$  and  $B$  be on a circle with radius 6 and center  $O$ . If  $\angle AOB = 90^\circ$ , find the area of triangle  $AOB$ .

*Proposed by Bradley Guo.*

**Answer:**  $\boxed{18}$

**Solution:** Triangle  $AOB$  is just a right triangle with side lengths 6 and 6. The area of a right triangle is half the product of its side lengths, which in this case is 18.

- 6 A scalene triangle (the 3 side lengths are all different) has integer angle measures (in degrees). What is the largest possible difference between two angles in the triangle?

*Proposed by Bradley Guo.*

**Answer:**  $\boxed{176}$

**Solution:** A scalene triangle has three different angles all summing to 180. The largest possible difference between two of these angles happens in a triangle with angles  $1^\circ$ ,  $2^\circ$ , and  $177^\circ$ . The difference is  $177 - 1 = 176$ .

- 7 Square  $ABCD$  has side length 6. If triangle  $ABE$  has area 9, find the sum of all possible values of the distance from  $E$  to line  $CD$ .

*Proposed by Bradley Guo.*

**Answer:**  $\boxed{12}$

**Solution:** The distance from  $E$  to line  $AB$  is the height of triangle  $ABE$  which must be 3. Point  $E$  can either be on the same side of  $AB$  as  $CD$  or on the opposite side. The distance from  $E$  to  $CD$  is 3 and 9 respectively. Their sum is 12.

- 8 Let point  $E$  be on side  $\overline{AB}$  of square  $ABCD$  with side length 2. Given  $DE = BC + BE$ , find  $BE$ .

*Proposed by Bradley Guo.*

**Answer:**  $\boxed{\frac{1}{2}}$

**Solution:** Let  $BE = x$ . Then  $AE = 2 - x$  and  $DE = 2 + x$ . Applying the Pythagorean Theorem gives

$$2^2 + (2 - x)^2 = (2 + x)^2$$

Expanding and solving for  $x$  gives  $x = \frac{1}{2}$