

Solutions to Dedekind Counting and Probability

- 1 Burgerly flips 2 fair coins. What is the probability that the second coin lands tails?

Proposed by Nathan Shan.

Answer: $\boxed{\frac{1}{2}}$

Solution: The first coin doesn't affect the probability of coin landing tails, and the coin is fair, thus this probability is $\frac{1}{2}$

- 2 Jooby has a nickel, a dime, and a quarter. If he loses a random coin, what is the probability that he has at least 30 cents left over?

Proposed by Bradley Guo.

Answer: $\boxed{\frac{2}{3}}$

Solution: We see that the value of a nickel and a quarter or a dime and a quarter is greater than 30 cents, while a nickel and a dime is less. The probability of not getting only the nickel and the dime is $\frac{2}{3}$.

- 3 Two identical caps each have 4 paper slips with numbers 1, 2, 4, and 8 written on them. Madeline takes out one paper slip from each cap, multiplies the two numbers she sees, and buys that number of strawberries. How many different values are possible for the number of strawberries that she will buy?

Proposed by Bradley Guo and Justin Chen.

Answer: $\boxed{7}$

Solution: Notice that all these numbers are powers of 2, so the only products we can get are also powers of 2. The smallest product is 1 and the largest is 64, so the possible values are 1, 2, 4, 8, 16, 32, 64 and there are 7 of them.

- 4 Two different positive integers sum to 10. How many possibilities are there for their product?

Proposed by Bradley Guo.

Answer: $\boxed{4}$

Solution: We let the minimum of the two different positive integers be a , then the other integer has to be $10 - a$ so the product will be $a(10 - a)$. Since the two integers are different, a cannot be 5. We know that a can range from 1 to 4 and each value of a gives a different product, thus there are 4 possibilities total.

- 5 Two girls play rock-paper-scissors, where each side throws out a rock, a paper, or a scissors with equal probability. They tie if they throw out the same move. What is the probability that the two girls tie?

Proposed by Bradley Guo.

Answer: $\boxed{\frac{1}{3}}$

Solution: No matter what the first girl throws out, the probability that the second girl matches her to get a tie is $\frac{1}{3}$, so the overall probability is $\frac{1}{3}$.

- 6 A triangle has sides of length 2, a square has sides of length 3, and a pentagon has sides of length 4. Two sides are chosen from the 12 sides. What is the probability that the two chosen sides have the same length?

Proposed by Bradley Guo.

Answer: $\boxed{\frac{19}{66}}$

Solution: We can either have two sides from the triangle, two sides from the square, or two sides from the pentagon. The probability of the first case is $\frac{\binom{3}{2}}{\binom{12}{2}}$, the probability of the second is $\frac{\binom{4}{2}}{\binom{12}{2}}$, and the probability of the third is $\frac{\binom{5}{2}}{\binom{12}{2}}$. Adding gives $\frac{3+6+10}{66} = \frac{19}{66}$.

- 7 Zorian rolls two fair 6-sided dice and multiplies their results. What is the probability that his final number ends in a 0?

Proposed by Bradley Guo.

Answer: $\boxed{\frac{1}{6}}$

Solution: For the final number to end in a 0, the product must be divisible by 5, so one of the dice results must be divisible by 5 and thus is 5, and then the other dice must be even. The probability that the first dice is 5 and the second is even is $\frac{1}{12}$, and we get the same probability if the second dice is 5 and the first is even, so the total probability is $\frac{2}{12} = \frac{1}{6}$.

- 8 Gose shuffles a standard deck of 52 cards. He flips over the first three cards: 8, 2, and 3. What is the probability that when he flips over the fourth card, the sum of the four values is greater than 21? Jacks, queens, and kings have a value of 10, and aces have a value of 11.

Proposed by Bradley Guo.

Answer: $\boxed{\frac{24}{49}}$

Solution: For the sum of the four values to be greater than 21, then the fourth card must be greater than $21 - 8 - 2 - 3 = 8$. Thus, it must be a 9, 10, jack, queen, king, or ace. There are 4 of each of these rank of card in the deck, so there are 24 possible winning cards. The deck has 49 cards left since the first 3 were removed, so the probability is $\frac{24}{49}$.