

Solutions to Dedekind Algebra

- 1 Bradley loves to eat bread. Every day he cuts a 1 centimeter wide slice of bread for breakfast. If a single loaf of bread is 25 centimeters long, how many weeks would 7 loafs of bread last him?

Proposed by River Chen.

Answer: $\boxed{25}$

Solution: Each loaf provides 25 slices of 1 centimeter wide bread, which means each loaf lasts for 25 days. Seven loaves will last $7 \cdot 25 = 175$ days, which is 25 weeks.

- 2 Jon is on an escalator. He is currently two thirds of the way up. If the escalator is 15 feet tall, how many feet are between him and the ground?

Proposed by Nathan Shan.

Answer: $\boxed{10}$

Solution: Since he is two thirds the way up, the distance to the ground is $\frac{2}{3} \cdot 15 = 10$.

- 3 Michelle finds a website which gives her a free textbook every day. After gaining 10 textbooks in 10 days, she has tripled her textbook collection. How many textbooks does Michelle have 20 days after she found the website?

Proposed by Nathan Shan.

Answer: $\boxed{25}$

Solution: If Michelle starts with x textbooks, then we have that $x + 10 = 3x$, and moving the x to one side gives us that $2x = 10$, or equivalently, that $x = 5$. Therefore, in 20 days, Michelle will have $x + 20 = 25$ books.

- 4 Steven really likes palindromes. Palindromes are numbers that read the same backwards and forwards, like 55 or 969. He's thinking of a 3 digit palindrome where the sum of digits is 16 and the ones digit is 5 more than the tens digit. What is the number?

Proposed by Steven Wang.

Answer: $\boxed{727}$

Solution: The palindrome must have the same hundreds digit and units digit. We can write this as $\overline{aba} = 100a + 10b + a$, where a and b are digits. The problem tells us that $a + b + a = 16$ and $a = b + 5$. Substituting and collecting like terms, we get $3b + 10 = 16$, or $b = 2$.

Finally, we remember that $a = b + 5$, so $a = 7$.

- 5 Bob has a magic trick. He claims that you can give him any number. Then, if he adds 2, multiplies by n , subtracts 6, then divides by n , he gets the original number back. What is n ?

Proposed by Kevin Wu.

Answer: $\boxed{3}$

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Solution: Let x be the number that you give. Bob will then find $\frac{n(x+2)-6}{n}$, which will be equal to n . Expanding gives that

$$\frac{nx + 2n - 6}{n} = \frac{nx}{n} + \frac{2n - 6}{n} = x + \frac{2n - 6}{n} = x.$$

For this to be true, $2n - 6$ has to be zero. Therefore, $n = 3$.

- 6 Gablin and Babblin start with different amounts of grapes. If Gablin gives Babblin 1 grape, Gablin would have the number of grapes Babblin has, squared. If Babblin gave Gablin 1 grape instead, Gablin would have had the number of grapes Babblin has, cubed. How many grapes does Gablin have?

Proposed by Bradley Guo.

Answer: 26

Solution: Let the number of grapes that Gablin has be x , and the number of grapes Babblin has be y .

The problem tells us that $x - 1 = (y + 1)^2$, and that $x + 1 = (y - 1)^3$.

Since $(y - 1)^3$ grows much faster than $(y + 1)^2$, we know that y is small. We find $y = 4$ by checking the first few values of y , and thus, $x = 26$.

- 7 Suppose that x and y are nonzero real numbers that satisfy $3x^2 = 4y^2$. What is the product of all possible values of $\frac{x+y}{x-y}$?

Proposed by Bradley Guo.

Answer: 1

Solution: From the equation we get $y = \pm \frac{\sqrt{3}}{2}x$. The product of the possible values of $\frac{x+y}{x-y}$ are

$$\frac{x + \frac{\sqrt{3}}{2}x}{x - \frac{\sqrt{3}}{2}x} \cdot \frac{x - \frac{\sqrt{3}}{2}x}{x + \frac{\sqrt{3}}{2}x}$$

which is 1 since they are reciprocals of each other.

- 8 Find the number of integers whose nearest perfect square is 264^2 , including 264^2 itself.

Proposed by Bradley Guo.

Answer: 528

Solution: There are $264^2 - 263^2 - 1 = (264 - 263)(264 + 263) - 1 = 526$ numbers between 264^2 and 263^2 , exclusive.

There are $265^2 - 264^2 - 1 = (265 - 264)(265 + 264) - 1 = 528$ numbers between 264^2 and 265^2 , exclusive.

For each of the two groups of numbers, exactly half of them are closer to 264^2 .

Including 264^2 , we get a final answer of $\frac{526}{2} + \frac{528}{2} + 1 = 528$.